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Global Regularity for the 2D Magneto-Micropolar Equations with Partial Dissipation

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Abstract. This paper studies the global existence and regularity of classical solutions to the 2D incompressible magneto-micropolar equations with partial dissipation. The magneto-micropolar equations model the motion of electrically conducting micropolar fluids in the presence of a magnetic field. When there is only partial dissipation, the global regularity problem can be quite difficult. We are able to single out three special partial dissipation cases and establish the global regularity for each case. As special consequences, the 2D Navier-Stokes equations, the 2D magnetohydrodynamic equations, and the 2D micropolar equations with several types of partial dissipation always possess global classical solutions. The proofs of our main results rely on anisotropic Sobolev type inequalities and suitable combination and cancellation of terms.

AMS subject classifications: 35Q35, 35B35, 35B65, 76D03

Key words: Global regularity, magneto-micropolar equations, partial dissipation.

1 Introduction

This paper aims at the global existence and regularity of classical solutions to the 2D incompressible magneto-micropolar equations with partial dissipation. The standard 3D incompressible magneto-micropolar equations can be written as

$$\begin{cases} \partial_t u+(u\cdot\nabla)u+\nabla(p+\frac{1}{2}|b|^2)=(\mu+\chi)\Delta u+(b\cdot\nabla)b+2\chi\nabla\times\omega, \\ \partial_t b+(u\cdot\nabla)b=\nu\Delta b+(b\cdot\nabla)u, \\ \partial_t\omega+(u\cdot\nabla)\omega+2\chi\omega=\kappa\Delta\omega+(\alpha+\beta)\nabla\nabla\cdot\omega+2\chi\nabla\times u, \\ \nabla\cdot u=0, \quad \nabla\cdot b=0, \end{cases} \tag{1.1}$$

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where, for $\mathbf{x}\in\mathbb{R}^3$ and $t\geq 0$, $u=u(\mathbf{x},t), b=b(\mathbf{x},t), \omega=\omega(\mathbf{x},t)$ and $p=p(\mathbf{x},t)$ denote the velocity field, the magnetic field, the micro-rotation field and the pressure, respectively, and μ denotes the kinematic viscosity, ν the magnetic diffusivity, χ the vortex viscosity, and α, and β and κ the angular viscosities. The 3D magneto-micropolar equations reduce to the 2D magneto-micropolar equations when

$$\begin{aligned}&u=(u_1(x,y,t),u_2(x,y,t),0), \quad b=(b_1(x,y,t),b_2(x,y,t),0),\\ &\omega=(0,0,\omega(x,y,t)), \quad \pi=\pi(x,y,t),\end{aligned}$$

where $(x,y)\in\mathbb{R}^2$ and we have written $\pi=p+\frac{1}{2}|b|^2$. More explicitly, the 2D magneto-micropolar equations can be written as

$$\begin{cases}\partial_t u+(u\cdot\nabla)u+\nabla\pi=(\mu+\chi)\Delta u+(b\cdot\nabla)b+2\chi\nabla\times\omega,\\ \partial_t b+(u\cdot\nabla)b=\nu\Delta b+(b\cdot\nabla)u,\\ \partial_t\omega+(u\cdot\nabla)\omega+2\chi\omega=\kappa\Delta\omega+2\chi\nabla\times u,\\ \nabla\cdot u=0, \quad \nabla\cdot b=0,\end{cases} \tag{1.2}$$

where $u=(u_1,u_2)$, $b=(b_1,b_2)$, $\nabla\times\omega=(-\partial_y\omega,\partial_x\omega)$ and $\nabla\times u=\partial_x u_2-\partial_y u_1$.

The magneto-micropolar equations model the motion of electrically conducting micropolar fluids in the presence of a magnetic field. Micropolar fluids represent a class of fluids with nonsymmetric stress tensor (called polar fluids) such as fluids consisting of suspending particles, dumbbell molecules, etc (see, e.g., [6,8–10,17]). A generalization of the 2D magneto-micropolar equations is given by

$$\begin{cases}\partial_t u_1+(u\cdot\nabla)u_1+\partial_x\pi=\mu_{11}\partial_{xx}u_1+\mu_{12}\partial_{yy}u_1+(b\cdot\nabla)b_1-2\chi\partial_y\omega,\\ \partial_t u_2+(u\cdot\nabla)u_2+\partial_y\pi=\mu_{21}\partial_{xx}u_2+\mu_{22}\partial_{yy}u_2+(b\cdot\nabla)b_2+2\chi\partial_x\omega,\\ \partial_t b+(u\cdot\nabla)b=\nu_1\partial_{xx}b+\nu_2\partial_{yy}b+(b\cdot\nabla)u,\\ \partial_t\omega+(u\cdot\nabla)\omega+2\chi\omega=\kappa_1\partial_{xx}\omega+\kappa_2\partial_{yy}\omega+2\chi\nabla\times u,\\ \nabla\cdot u=0, \quad \nabla\cdot b=0,\\ u(x,y,0)=u_0(x,y), b(x,y,0)=b_0(x,y), \omega(x,y,0)=\omega_0(x,y),\end{cases} \tag{1.3}$$

where we have written the velocity equation in its two components. Clearly, if

$$\mu_{11}=\mu_{12}=\mu_{21}=\mu_{22}=\mu+\chi, \quad \nu_1=\nu_2=\nu, \quad \kappa_1=\kappa_2=\kappa,$$

then (1.3) reduces to the standard 2D magneto-micropolar equations in (1.2). This generalization is capable of modeling the motion of anisotropic fluids for which the diffusion properties in different directions are different. In addition, (1.3) allows us to explore the smoothing effects of various partial dissipations.

The magneto-micropolar equations above are not only important in engineering and physics, but also mathematically significant. The mathematical study of the magneto-micropolar equations started in the seventies and has been continued by many authors