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POD Applied to Numerical Study of Unsteady Flow Inside Lid-driven Cavity

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Abstract. Flow inside a lid-driven cavity (LDC) is studied here to elucidate bifurcation sequences of the flow at super-critical Reynolds numbers (Re_{cr1}) with the help of analyzing the time series at most energetic points in the flow domain. The implication of Re_{cr1} in the context of direct simulation of Navier-Stokes equation is presented here for LDC, with or without explicit excitation inside the LDC. This is aided further by performing detailed enstrophy-based proper orthogonal decomposition (POD) of the flow field. The flow has been computed by an accurate numerical method for two different uniform grids. POD of results of these two grids help us understand the receptivity aspects of the flow field, which give rise to the computed bifurcation sequences by understanding the similarity and differences of these two sets of computations. We show that POD modes help one understand the primary and secondary instabilities noted during the bifurcation sequences.

AMS subject classifications: 65M12, 65M15, 65M60, 76D05, 76F20, 76F65

Key words: Lid driven cavity, POD, POD modes analysis, DNS, multiple Hopf bifurcation, polygonal core vortex.

1 Introduction

The 2D flow in a square LDC (of side L) is a canonical problem to study flow dynamics numerically for incompressible Navier-Stokes equation due to its unambiguous boundary conditions and very simple geometry. The flow is essentially shear-driven, with the lid given a constant-speed translation (U), giving rise to corner singularities on the top

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wall, as depicted in the top frame of Figure 1. Such singularity gives rise to Gibbs' phenomenon [1, 5], which is milder for low order methods [16, 29]. Low order highly diffusive methods [6,16] are incapable of computing unsteady flows at high Reynolds number ($Re{=}UL/\nu$, where ν is the kinematic viscosity). In Ghia *et al.* [16], results for a wide range of Re up to 10000 are presented as steady flow. However, numerical results obtained by high accuracy combined compact difference scheme indicate creation of a transient polygonal vortex at the core, with permanent gyrating satellite vortices around it [38,42], for the same Re. It is well known that compact schemes for spatial discretization behave properly as compared to other methods, and Gibbs' phenomenon [35] is not experienced for the singular LDC problem due to numerical smoothing of the derivatives near the Nyquist limit [31, 39].

Steady solutions have been reported [14, 16] for Re far exceeding the values reported in the literature for the first Hopf Bifurcation (Re_{cr1}). Unsteady flows have been obtained as a solution of bifurcation problem [26, 43], by studying linear temporal instability of the steady solution obtained numerically. Simulations of full time-dependent Navier-Stokes equation [25, 38] reveal that the flow loses stability via Hopf bifurcation, as Re increases. Critical Re and frequencies obtained from DNS and eigenvalue analysis do not match. Such differences are also noted for different DNS results. However, DNS approach is preferable, due to its superiority of spatio-temporal multi-modal analysis over normal mode analysis of eigenvalue approach. In the latter, one postulates explicitly that all points in the domain have identical variation with respect to time. This is strictly incorrect, as one is dealing with space-time dependent growth of disturbances during the onset of unsteadiness.

It is shown [25, 41, 42] that Re_{cr1} depends upon accuracy of the method and how the flow is established in DNS. Impulsive start of the flow triggers all frequencies at the onset and hence preferred [38, 42]. Obtaining final limit cycle at one Re from the limit cycle solution from another Re [25] is inappropriate [22]. First Hopf bifurcation obtained by DNS is dependent upon source of numerical error, mainly on the aliasing error for flow inside LDC [42]. This also depends upon the discretization, which in turn determines the creation of wall vorticity. A finer grid will create larger wall vorticity, but will have lesser truncation error. For the same numerical method, using same time step, a finer grid will have lesser aliasing and truncation errors, and hence numerical Re_{cr1} will be higher for finer grid. However, this can also be studied with the help of explicit excitation to show the near universality of Re_{cr1}.

Linear instability of equilibrium flow and DNS have been used to evaluate the onset of unsteadiness, i.e., obtaining Re_{cr1} for LDC. These methods yield values of Re_{cr1} differently. For example, $Re_{cr1} = 8018$ in [2] and 8031.93 in [28] have been reported. Cazemier *et al.* [8] reported Re_{cr1} at 7972 using a finite volume method. In Bruneau and Saad [6], the critical Re is suggested to be in the range of $8000 \leq Re_{cr1} \leq 8050$, obtained using a third order upwind finite difference scheme. The authors do not provide any bifurcation diagram to substantiate this observation. Sengupta *et al.* [41] have described multiple Hopf bifurcations, showing the first one at 7933 and the second at 8187, using uniform