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Research on Equivalence of SVD and PCA in Medical Image Tilt Correction ⋆

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Abstract

In the process of medical imaging, often because of some disturbance, the medical images frequently have some undesirable tilt, which has costly negative effect on the following image alignment and fusion. In order to solve the tilt problem, Singular Value Decomposition (SVD) and Principal Component Analysis (PCA) are studied and their relationship between them is discussed, and then the medical image correction tilt process is divided into five main stages. Among these stages, the key tasks focus on finding the centroid and obtaining the tilt angle of a medical image. We use SVD and PCA to compute the eigenvectors of the coordinates of a medical image respectively to get the tilt angle. The experimental results reveal that the methods mentioned above are effective for correcting the tilt medical images and also prove the equivalence of SVD and PCA in medical image tilt correction.

Keywords: Equivalence; SVD; PCA; Tilt Correction

1 Introduction

When the computed tomography and magnetic resonance images are generated, often due to the pertinent imaging device with a low stability, the patients with voluntary movement or with emotional tension, the medical images often have some outrageous tilt, which has costly negative effect on the subsequent image alignment and fusion, and even misdiagnoses of the diseases. Also, the tilt correction is the crucial step in the field of medical image segmentation because it largely and directly determines the segmentation results such as the extraction of the contour and focus of a disease and the shape features. Therefore, it is very necessary to correct the tilt.

In the field of image tilt correction, such as in the tilt document image and the tilt vehicle license plate image, many researchers around the world have made beneficial exploration and offered us abundant and creative tilt correction methods. Among these methods, the most typical

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and prevalent are the following two kinds: Projection Profile (PP) [1] and Hough Transformation (HT) [2]. They are very adapted for dealing with the uniform gray or binarization tilt images such as the tilt document image and the tilt vehicle license plate image, but not for the nonuniform gray images, especially such as tilt medical images. Therefore, we have to explore some new methods to correct the tilt medical images.

So far, few published sources or literatures especially catering to the tilt issue in the context of medical images have been mentioned and introduced. On the foundation of a comprehensive and thorough research into Singular Value Decomposition (SVD) and Principal Component Analysis (PCA), we find that the eigenvectors of the covariance matrix of PCA can be derived by the use of SVD, so we use the equivalence of SVD and PCA to correct the medical image tilt.

2 SVD and PCA

2.1 SVD

Singular value decomposition, which, as one of the modern numerical analysis tools, is an important matrix decomposition technology in the area of linear algebra and is the generalization of unitary diagonalization of a normal matrix in matrix analysis, serves as an extremely important role in the fields of signal processing, statistics and so on, and is often also used for image processing, such as the image restoration by singular value decomposition [3], quantitative mapping of structured polymeric systems using singular value decomposition analysis of soft X-ray images [4], adaptive MRI with encoding by singular value decomposition [5], and so on.

Suppose that $\boldsymbol{A} \in R_r^{m\times n}(r > 0)$, where $R_r^{m\times n}$ denotes a set of $m \times n$ real matrices with rank r. For the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_r, \lambda_{r+1}, \cdots, \lambda_n$ of $\boldsymbol{A}^T\boldsymbol{A}$ where $\boldsymbol{A}^T$ represents the transposed matrix of $\boldsymbol{A}$, we have $\lambda_1 \geq \lambda_2 \geq \cdots > \lambda_r > \lambda_{r+1} = \cdots = \lambda_n = 0$, then $\sigma_i = \sqrt{\lambda_i}(i = 1, 2, \cdots, n)$ is called the singular value of matrix $\boldsymbol{A}$. Especially, in the case of $\boldsymbol{A} = \boldsymbol{0}$, all its singular values are 0. Therefore, the number of the singular values of matrix $\boldsymbol{A}$ is equal to that of the columns of matrix $\boldsymbol{A}$, and the number of the nonzero singular values of matrix $\boldsymbol{A}$ is equal to the rank of matrix $\boldsymbol{A}$.

Theorem 1 Let $\boldsymbol{A} \in R_r^{m\times n}(r > 0)$, then there exist a $m \times m$ matrix $\boldsymbol{U}$ and a $n \times n$ matrix $\boldsymbol{V}$ satisfying

$$\boldsymbol{U}^T\boldsymbol{A}\,\boldsymbol{V} = \begin{bmatrix} \boldsymbol{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} \tag{1}$$

where $\boldsymbol{\Sigma} = diag(\sigma_1, \sigma_2, \cdots, \sigma_r)$ and $\sigma_i(i = 1, 2, \cdots, r)$ expresses all the nonzero singular values of matrix $\boldsymbol{A}$. Eq. (1) can be rewritten as

$$\boldsymbol{A} = \boldsymbol{U} \begin{bmatrix} \boldsymbol{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{V}^T \tag{2}$$

Eq. (2) is termed the singular values of matrix $\boldsymbol{A}$.

Assume that $\boldsymbol{A} \in R^{2\times N}$ is nonsingular, i.e., $\boldsymbol{A}$ is a set of vectors in the two-dimensional real space, whose elements can be marked with $[\boldsymbol{X}\ \boldsymbol{Y}]^T$, where $\boldsymbol{X} = [x_1, x_2, \cdots, x_N]^T$ and $\boldsymbol{Y} = [y_1, y_2, \cdots, y_N]^T$.