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HIGH ORDER COMPACT MULTISYMPLECTIC SCHEME FOR COUPLED NONLINEAR SCHRÖDINGER-KDV EQUATIONS*

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Abstract

In this paper, a novel multisymplectic scheme is proposed for the coupled nonlinear Schrödinger-KdV (CNLS-KdV) equations. The CNLS-KdV equations are rewritten into the multisymplectic Hamiltonian form by introducing some canonical momenta. To simulate the problem efficiently, the CNLS-KdV equations are approximated by a high order compact method in space which preserves N semi-discrete multisymplectic conservation laws. We then discretize the semi-discrete system by using a symplectic midpoint scheme in time. Thus, a full-discrete multisymplectic scheme is obtained for the CNLS-KdV equations. The conservation laws of the full-discrete scheme are analyzed. Some numerical experiments are presented to further verify the convergence and conservation laws of the new scheme.

Mathematics subject classification: 65M06, 65M12, 65Z05, 70H15
Key words: Schrödinger-KdV equations, High order compact method, Conservation law, Multisymplectic scheme.

1. Introduction

Multisymplectic schemes are very popular in numerical computing context for Hamiltonian PDEs since the end of last century due to the observation of their excellent behavior in long time [2, 6, 9–11, 14, 17, 19]. The most frequently used approaches to construct multisymplectic schemes are concatenating methods [24], for example, method of concatenating a pair of Runge-Kutta methods in space and in time respectively, method of concatenating a symplectic Runge-Kutta method in time with a spectral method or a finite element method in space [5,12,13,18,21]. In this paper, we attempt to construct a high order compact (HOC) multisymplectic scheme by concatenating a symplectic Runge-Kutta method in time with a high order compact method in space. Meanwhile, the high order compact multisymplectic scheme is applied to the coupled

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nonlinear Schrödinger-KdV (CNLS-KdV) equations [4, 26]

$$iU_t + U_{xx} - UV = 0, \quad i^2 = -1, \qquad x \in \mathbb{R},\ t > 0, \tag{1.1}$$

$$V_t + \beta V_{xxx} + \frac{1}{2}\alpha(V^2)_x - (|U|^2)_x = 0, \qquad x \in \mathbb{R},\ t > 0, \tag{1.2}$$

where $U(x,t)$ and $V(x,t)$ denote the complex-valued amplitude of the short wave and real-valued amplitude of the long wave, respectively, α and β are the nonlinear and dispersive parameters, respectively. This model describes the influence between longitudinal wave $V(x,t)$ and slowly varying envelopes of the short transverse wave $U(x,t)$ in dispersive media.

We consider the CNLS-KdV equations (1.1)-(1.2) with the following initial-boundary conditions

$$\lim_{|x|\to\infty} U(x,t) = 0, \qquad \lim_{|x|\to\infty} V(x,t) = 0, \tag{1.3}$$

$$U(x,0) = U_0(x), \qquad V(x,0) = V_0(x). \tag{1.4}$$

Then we can easily prove that the CNLS-KdV equations (1.1)-(1.4) follow the conserved quantities:

$$\mathcal{P}(t) = \int_{\mathbb{R}} V(x,t)dx = \int_{\mathbb{R}} V_0(x)dx = \mathcal{P}(0), \tag{1.5}$$

$$\mathcal{D}(t) = \int_{\mathbb{R}} |U(x,t)|^2 dx = \int_{R} |U_0(x)|^2 dx = \mathcal{D}(0), \tag{1.6}$$

$$\begin{aligned}\mathcal{H}(t) &= \int_{\mathbb{R}} (|U_x(x,t)|^2 + V(x,t)|U(x,t)|^2 + \frac{\beta}{2}(V_x(x,t))^2 - \frac{\alpha}{6}(V(x,t))^3)dx \\ &= \int_{\mathbb{R}} (|U_{0x}(x)|^2 + V_0(x)|U_0(x)|^2 + \frac{\beta}{2}(V_{0x}(x))^2 - \frac{\alpha}{6}(V_0(x))^3)dx \\ &= \mathcal{H}(0),\end{aligned} \tag{1.7}$$

$$\begin{aligned}\mathcal{M}(t) &= \int_{\mathbb{R}} [\Im(U(x,t)\overline{U_x(x,t)}) + \frac{1}{2}(V(x,t))^2]dx \\ &= \int_{\mathbb{R}} [\Im(U_0(x)\overline{U_{0x}(x)}) + \frac{1}{2}(V_0(x))^2]dx \\ &= \mathcal{M}(0),\end{aligned} \tag{1.8}$$

where $\Im$ denotes taking imaginary part. The first invariant (1.5) implies that the number of particles is unchanged all along. The second one (1.6) is the so-called invariant of mass. The third one (1.7) and the last one (1.8) are respectively the general conservation laws of the Hamiltonian energy and momentum in the closing dynamic system. For the detailed proof of the conservation laws, we refer to [27].

Numerical methods with high order accuracy are desired. For this purpose, one usually widens the stencil to approximate derivatives. That is, one uses more adjacent nodes to get higher accuracy. This obviously increases the complexity in practical computing. The typical case is the spectral method which is of exponential convergence rate for sufficiently smooth problems, but it results in full spectral differentiation matrices. To obtain high accuracy, an alternative approach is the so-called HOC method [15, 16, 22]. Such kind of approximation is not only of high accuracy, but also of small numerical dissipative errors and dispersive errors. It features both high order accuracy and small stencil which leads to narrow bandwidth matrices.

The CNLS-KdV equations have been analyzed qualitatively by some authors [1, 4, 8, 20, 25] ever since the pioneering works of Tsutsumi and Hatano [20]. There were some numerical investigations on the CNLS-KdV equations [3, 7, 8]. Bai et al [3] developed a B-spline finite