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AN AUGMENTED LAGRANGIAN TRUST REGION METHOD WITH A BI-OBJECT STRATEGY*

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Abstract

An augmented Lagrangian trust region method with a bi-object strategy is proposed for solving nonlinear equality constrained optimization, which falls in between penalty-type methods and penalty-free ones. At each iteration, a trial step is computed by minimizing a quadratic approximation model to the augmented Lagrangian function within a trust region. The model is a standard trust region subproblem for unconstrained optimization and hence can efficiently be solved by many existing methods. To choose the penalty parameter, an auxiliary trust region subproblem is introduced related to the constraint violation. It turns out that the penalty parameter need not be monotonically increasing and will not tend to infinity. A bi-object strategy, which is related to the objective function and the measure of constraint violation, is utilized to decide whether the trial step will be accepted or not. Global convergence of the method is established under mild assumptions. Numerical experiments are made, which illustrate the efficiency of the algorithm on various difficult situations.

Mathematics subject classification: 90C55, 65K05, 90C30
Key words: Nonlinear constrained optimization; Augmented Lagrangian function; Bi-object strategy; Global convergence.

1. Introduction

The augmented Lagrangian (AL) method has become a class of important methods for constrained optimization [3] since its proposition by Hestenes [17] and Powell [22]. Specifically, Conn et al. [11] presented a practical augmented Lagrangian method, based on which a well-known package Lancelot [12] was released. The attractive features of AL methods are that, they can be implemented matrix-free [4,11] and possess fast local convergence guarantees under relatively weak assumptions [1]. Moreover, AL methods have successfully been applied in many

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fields such as compressed sensing [2], distributed optimization and statistical learning [6] and signal reconstruction [25].

One efficient way to implement the AL method is to minimize some approximation model to the augmented Lagrangian function with some trust region at each iteration. On the other hand, it is known that if directly adding the trust region to the linearized constraints, this may lead to inconsistency (for example). To tackle the inconsistency, one may incorporates the linearized constraints into the objective function by virtue of some penalty factor, such as the l_1 exact penalty [13], the l_∞ exact penalty [26] and the l_2 exact penalty [20, 24]. The algorithms in these works can be regarded variants of the AL method. Meanwhile, we can also see that the AL method with the trust region technique circumvents the difficulty of dealing the inconsistency between the linearized constraints and the trust region. It should be noticed here that the use of the l_2 exact penalty will bring standard single-ball trust region subproblems for unconstrained optimization and hence can be solved efficiently by many mature methods.

Further, exact penalty methods, including the AL method, have proved to be effective techniques for solving difficult nonlinear programs. They are successful in solving certain classes of problems, in which standard constraint qualifications are not satisfied [7, 8]. However, there is still some trouble in choosing appropriate values of the penalty parameter. The performance of the AL method suffers when the choice of the penalty parameter and/or Lagrange multipliers is very poor. If the penalty parameter is too large and/or the estimate to the true Lagrangian multipliers is bad, there might be little or no progress in the primal space due to the iterates veering too far away from the feasible region. It is also not suitable to calculate a solution with higher precision if the choice of the penalty parameter and/or Lagrange multipliers is poor. Moreover, the penalty parameter in various approaches proposed in the literature may tend to infinity [7, 24]. Take the following infeasible problem as an example.

$$\begin{array}{ll} \min & (x_1-1)^2 \\ \text{s.t.} & x_2^2+1=0. \end{array} \tag{1.1}$$

It is easy to see that $x^* = (1,0)^T$ is an infeasible stationary point. The augmented Lagrangian function of problem (1.1) is

$$P(x,\lambda,\sigma) = (x_1-1)^2 - \lambda(x_2^2+1) + \frac{1}{2}\sigma(x_2^2+1)^2.$$

The AL method aims to solve a sequence of unconstrained problems, $\min P(x,\lambda_k,\sigma_k)$, with $\{\sigma_k\}$, $\{\lambda_k\}$ determined in some way. If $\sigma_k > |\lambda_k|$, the augmented Lagrangian function $P(x,\lambda_k,\sigma_k)$ has a unique stationary point $x_k(\lambda_k,\sigma_k) = x^*$. If this happens, we see that the measure of the constraint violation remains unchanged; *i.e.*, $|c(x_k(\lambda_k,\sigma_k))| \equiv 1$. Thus by the mechanism of the AL method, the penalty parameter will be augmented ceaselessly until numerical overflows occur.

The above example shows that the penalty parameter in the AL method may monotonically tend to infinity and result in numerical overflows. It is known that the purpose of the penalization is to enforce the iterates approaching the feasible region. When the iterate is far away from the feasible region, it sounds reasonable to increase the penalty parameter to improve the feasibility. However, when the iterate becomes more and more feasible, we may think of reducing the penalty parameter. In other words, it is more reasonable to ask the penalty parameter to depend on the information about the current iterate. Based on this observation, we shall propose a new augmented Lagrangian trust region algorithm in this paper. The trial