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A BIE-BASED DtN-FEM FOR FLUID-SOLID INTERACTION PROBLEMS*

Tao Yin
College of Mathematics and Statistics, Chongqing University, Chongqing 401331, China
Email: taoyin_cqu@163.com
Andreas Rathsfeld
Weierstrass Institute, Mohrenstr. 39, 10117 Berlin, Germany
Email: rathsfeld@wias-berlin.de
Liwei Xu
College of Mathematics and Statistics, Chongqing University, Chongqing 401331, China,
Institute of Computing and Data Sciences, Chongqing University, Chongqing 400044, China
Email: xul@cqu.edu.cn

Abstract

In this paper, we are concerned with the coupling of finite element methods and boundary integral equation methods solving the classical fluid-solid interaction problem in two dimensions. The original transmission problem is reduced to an equivalent nonlocal boundary value problem via introducing a Dirichlet-to-Neumann mapping by the direct boundary integral equation method. We show the existence and uniqueness of the solution for the corresponding variational equation. Numerical results based on the finite element method coupled with the standard Galerkin boundary element method, the fast multipole method and the Nyström method for approximating the DtN mapping are provided to illustrate the efficiency and accuracy of the numerical schemes.

Mathematics subject classification: 65N38, 65N06, 74J05, 76B20.
Key words: Fluid-solid interaction problem, Dirichlet-to-Neumann mapping, Finite element method, Fast multipole method, Nyström method.

1. Introduction

Due to the considerable mathematical and computational challenges such as the oscillating character of solutions and the unbounded domain to be considered, the transmission problems involving acoustic waves scattered by a penetrable elastic body immersed in a fluid have been studied extensively for many years since the pioneering work by Faran [11]. These problems are of great importance in many fields of application including exploration seismology, oceanography, and non-destructive testing, to name a few. In this article, we are interested in the numerical solutions for the two dimensional fluid-solid interaction problem with bounded elastic structure, and we refer to [28] for the variational approach solving the fluid-solid interaction problem over periodic (bi-periodic) structures.

Recently, several numerical methods have been studied for the solution of the fluid-solid interaction problem including the boundary integral equation (BIE) method [31, 39] and its coupling with the finite element method (FEM) [8, 9, 16, 27, 33]. For the coupling scheme, a popular way is to use the BIE methods to solve the acoustic problem outside the obstacle while

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FEM is employed for the approximation of the interior elastic wave. It should be pointed out that any other field equation solver can also be used for solving the interior problem such as the discontinuous Galerkin method [3]. Another approach, the perfectly matched layer (PML) [2], to approximate free radiation is to introduce an additional damping layer surrounding the computational domain such that no reflections occur at its interface with the computational domain. This approach is easy to implement and is very effective. Another way to deal with the fact that the scattered acoustic wave propagates in an unbounded region is to introduce an artificial boundary enclosing the obstacle. Then, after imposing transparent boundary conditions [17] on the artificial boundary, we obtain a reduced nonlocal boundary value problem in a bounded domain which can be solved by field equation solvers. In particular, we can derive a Dirichlet-to-Neumann (DtN) mapping on the artificial boundary to obtain an exact transparent boundary condition, and accordingly this strategy is called DtN method [13]. The DtN mapping can be computed by boundary integral operators [18, 23, 32] or by Fourier-series expansions [12, 38]. The boundary integral equation based (BIE-based) DtN mapping can be defined on any smooth closed curve and this feature may reduce the size of the computational domain, while the Fourier expansion series based DtN mapping is usually defined on a circle or on a perturbation of a circle [34]. In this paper, we are interested in the BIE-based DtN-FEM, and we refer to [38] for the Fourier-series-based DtN-FEM solving the fluid-solid interaction problems.

In contrast to the methods, where the stress tensor is introduced as a main variable [15, 16], in this paper the displacement will be the unique unknown in the solid for the fluid-solid interaction problem [4, 27]. Following [32] but using a direct (cf. [26]) instead of an indirect way, we first introduce a nonlocal boundary value problem equivalent to the original problem by representing the exact DtN mapping via the boundary integral operators in acoutics, and then we investigate the existence and uniqueness of the solution for the corresponding variational equation. To compute the DtN mapping, we adopt the standard Galerkin boundary element method (SGBEM), the fast multipole method (FMM) [5,26,36] and the Nyström method (NM), respectively. Actually, if the number of those basis functions of the finite element space which do not vanish on the artificial boundary is N_θ, then we have to solve the BIE N_θ times with different Dirichlet boundary values as right-hand side to approximate the DtN mapping directly. However, following the techniques in [32] we observe that, if the artificial boundary is a circle, then it is sufficient to solve only one BIE with a fixed Dirichlet boundary value. Based on the solution of this BIE, we derive an explicit formulation to compute the sesquilinear form corresponding to the DtN mapping in this case.

The remainder of the paper is organized as follows. We first describe the classical fluid-solid interaction problem in Section 2, and then reduce the transmission problem to a nonlocal boundary value problem by defining a BIE-based DtN mapping in Section 3. Existence and uniqueness of the solution for the corresponding variational equation are discussed in Section 4. In Section 5, we describe the numerical schemes of the FEM and SGBEM, FMM, and NM for solving the boundary integral equation of the first kind. Finally, several numerical tests in Section 6 are presented to verify the efficiency and accuracy of the numerical schemes.

2. The Fluid-solid Interaction Problem

Let $\Omega \subset \mathbb{R}^2$ be an open, bounded, and simply connected domain with a closed and sufficiently smooth boundary $\Gamma = \partial\Omega$, and suppose its exterior complement is given by $\Omega^c = \mathbb{R}^2 \setminus \overline{\Omega} \subset$