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MEAN SQUARE STABILITY AND DISSIPATIVITY OF SPLIT-STEP THETA METHOD FOR NONLINEAR NEUTRAL STOCHASTIC DELAY DIFFERENTIAL EQUATIONS WITH POISSON JUMPS*

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Abstract

In this paper, a split-step θ (SST) method is introduced and used to solve the nonlinear neutral stochastic differential delay equations with Poisson jumps (NSDDEwPJ). The mean square asymptotic stability of the SST method for nonlinear neutral stochastic differential equations with Poisson jumps is studied. It is proved that under the one-sided Lipschitz condition and the linear growth condition, the SST method with $\theta \in (0, 2-\sqrt{2})$ is asymptotically mean square stable for all positive step sizes, and the SST method with $\theta \in (2-\sqrt{2}, 1)$ is asymptotically mean square stable for some step sizes. It is also proved in this paper that the SST method possesses a bounded absorbing set which is independent of initial data, and the mean square dissipativity of this method is also proved.

Mathematics subject classification: 65N06, 65B99.
Key words: Neutral stochastic delay differential equations, Split-step θ method, Stability, Poisson jumps.

1. Introduction

Stochastic functional differential equations (SFDEs) play important roles in science and engineering applications, especially for systems whose evolutions in time are influenced by random forces as well as their history information. When the time delays in SFDEs are constants, they turn into stochastic delay differential equations (SDDEs). Both the theory and numerical methods for SDDEs have been well developed in the recent decades, see[1–8]. Recently many dynamical systems not only depend on the present and the past states but also involve derivatives with delays, they are described as the neutral stochastic delay differential equations (NSDDEs). Compared to the stochastic functional differential equations and the stochastic delay differential equations, the study of the neutral stochastic delay differential equations has just started. In 1981, Kolmanovskii and Myshkis [8] took the environmental disturbances into account, introduced the NSDDEs and gave their applications in chemical engineering and aeroelasticity. The analytical solutions of NSDDEs are hardly to obtain, many authors have to study

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the numerical methods for NSDDEs, Wu and Mao [9] studied the convergence of the Euler-Maruyama method for neutral stochastic functional differential equations under the one-side Lipschitz conditions and the linear growth conditions. In 2009, Zhou and Wu [10] studied the convergence of the Euler-Maruyama method for NSDDEs with Markov switching under the one-side Lipschitz conditions and the linear growth conditions. The convergence of θ method and the mean square asymptotic stability of the semi-implicit Euler method for NSDDEs were studied in [11–14]. The convergence and mean square stability of split-step θ method and split-step backward Euler method for stochastic differential equations, stochastic delay differential equations and stochastic delay integro-differential equations were studied in [15–19]. The dissipativity and mean square stability of numerical methods for neutral stochastic delay differential equations were studied by Huang [20–22] and Zong et al. [23].

In addition, stochastic delay differential equations with Poisson jumps have become very popular in modeling the phenomena arising in the fields such as economics, physics, biology, medicine, and so on. Very recently, stochastic delay equations with Poisson jumps have attracted the interest of many researchers, see, e.g.,[24–28]. To the best of our knowledge, so far no work has been reported in the literature about NSDDEwPJ and this paper will close this gap.

The aim of this paper is to study the mean square stability and dissipativity of the split-step θ method with some conditions and the step constrained for NSDDEwPJ.

The paper is organized as follows: in section 2, some stability definitions about the analytic solutions for NSDDEwPJ are introduced, some notations and preliminaries are also presented in this section. In section 3, the split-step θ method is introduced and used to solve the NSDDEwPJ, the asymptotic stability of the split-step θ method is proved. In section 4, the long time behavior of numerical solution is studied and the mean-square dissipativity result of the method is illustrated. In Section 5, some numerical experiments are given to confirm the theoretical results.

2. Mean-square asymptotic stability of analytic solution

Let $|.|$ denotes both the Euclidean norm in R^d and the trace (or Frobenius) norm in $R^{d\times l}$ (denoted by$|A| = \sqrt{\mathrm{trace}(A^{\mathrm{T}}A)}$, if A is a vector or matrix, its transpose is denoted by A^{T}. Let $\{\Omega, F, (F_t)_{t\geq 0}, \mathrm{P}\}$ define a complete probability space with a filtration$\{F_t\}_{t\geq 0}$ which is increasing and right continuous, and F_0 contain all P-null sets. Let $w(t) = (w_1(t), w_2(t), \cdots, w_l(t))^{\mathrm{T}}$ denote standard l-dimensional Brownian motion on the probability space, and $w_i(t)$is the i-element of the l-dimention vector $w(t)$. In this paper we talk about the d-dimensional NSDDEs with Poisson jumps with the following form:

$$\begin{cases} \mathrm{d}(y(t) - \Psi(y(t-\tau))) = f(t, y(t), y(t-\tau))\mathrm{d}t + g(t, y(t), y(t-\tau))\mathrm{d}\omega(t) \\ \qquad\qquad\qquad\qquad\qquad + h(t, y(t), y(t-\tau))\mathrm{d}(N(t)), \quad t \geq 0, \\ y(t) = \phi(t), \quad t \in [-\tau, 0]. \end{cases} \tag{2.1}$$

where $\Psi : R^d \mapsto R^d$, $f : R_+ \times R^d \times R^d \mapsto R^d$, $g : R_+ \times R^d \times R^d \mapsto R^{d\times l}$ and $h : R_+ \times R^d \times R^d \mapsto R^d$ are the Borel measurable functions, τ is a positive constant delay, and $\phi(t)$ is an F_0 -measurable, $\mathrm{C}[-\tau, 0]; R^d)$ -valued random variable which satisfies

$$\sup_{-\tau\leq t\leq 0} \mathrm{E}[\phi^{\mathrm{T}}(t)\phi(t)] < \infty, \tag{2.2}$$