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DETECTION OF ELECTROMAGNETIC INCLUSIONS USING TOPOLOGICAL SENSITIVITY*

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Abstract

In this article, a topological sensitivity framework for far-field detection of a diametrically small electromagnetic inclusion is established. The cases of single and multiple measurements of the electric far-field scattering amplitude at a fixed frequency are taken into account. The performance of the algorithm is analyzed theoretically in terms of its resolution and sensitivity for locating an inclusion. The stability of the framework with respect to measurement and medium noises is discussed. Moreover, the quantitative results for signal-to-noise ratio are presented. A few numerical results are presented to illustrate the detection capabilities of the proposed framework with single and multiple measurements.

Mathematics subject classification: 35L05, 35R30, 74B05, 47A52, 65J20.
Key words: Electromagnetic imaging, Topological derivative, Localization, Resolution analysis, Stability analysis, Medium noise, Measurement noise.

1. Introduction

A thriving interest has been shown in topological sensitivity frameworks to procure solutions of assorted inverse problems, especially for detecting small inhomogeneities and cracks embedded in homogeneous media [2–5, 10, 13, 16–18, 21, 22, 26, 29–34]. The impetus behind this curiosity is the appositeness and simplicity of these algorithms. Notwithstanding their utility, the use of topological sensitivity based algorithms and their stability with respect to medium and measurement noises is more often heuristic and lacks rigorous quantitative analysis of resolution and signal-to-noise ratio.

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Ammari et al. [5] used asymptotic analysis to substantiate that the detection of small acoustic inclusions embedded in a bounded domain cannot be guaranteed without pre-processing the measurements using a *Calderón preconditioner* based on *Neumann-Poincaré* operator associated with the domain. Further, the detection of small inclusions nearly touching the boundary cannot be guaranteed. In [2], they established for linear isotropic elastic media that the mode-conversion and non-linear coupling of the wave modes at the boundary degenerate the localization and resolution of the classical imaging framework. Therefore, a weighted topological derivative based sensitivity framework was designed and debated for guaranteed inclusion detection. The stability and robustness of the acoustic and elastic algorithms were also debated in [2, 5]. In particular, it was proved for acoustic inclusion detection using multiple measurements that the topological derivative based imaging functions are more stable than contemporary techniques such as multiple signal classification (MUSIC), back-propagation and Kirchhoff migration. However, its computational complexity is relatively higher than the listed techniques. The analysis was further extended to the case of electromagnetic inclusion detection in [32] wherein the tangential components of the scattered magnetic field at the surface of a bounded domain were used. The results in [32] compliment those provided by Masmoudi et al. [26], wherein an adjoint field based approach is used to procure solutions of some inverse problems in non-destructive evaluation. The far-field imaging of acoustic inhomogeneities using topological sensitivity was discussed by Bellis et al. [10] using a factorization of the far-field operator. The cases of elastic inclusion detection using topological derivatives in half and full spaces were discussed by Bonnet and Guzina [12]. The topological sensitivity frameworks have been also developed to ascertain the morphology (shape, size, and material properties) of small inclusions and cracks. The interested readers are referred, for instance, to [9, 11, 21]. Moreover, a level set theory for a variety of location indicator functions was developed in [14] and [19]. The partial aperture problems and the computational aspects were addressed using multi-frequency approaches, for example, in [1, 20]. It is worthwhile preciseing that the topological sensitivity framework is primarily a *single-short* method and the multi-static configuration is generally used to enhance its performance in terms of stability and resolution. We refer, for instance, to [23–25] for other asymptotic frameworks for detection of multiple multiscale electromagnetic inclusions using single-short algorithms.

The goal in this investigation is to perform a quantitative analysis of a topological sensitivity functional to locate an electromagnetic inclusion of vanishing characteristic size using measurements of the far-field scattering amplitude of the electric field. The general case of an electric field satisfying full Maxwell equations is considered and the arguments are constructed using asymptotic analysis by Ammari and Volkov [8]. The stability with respect to medium and measurement noises is discussed and the estimates for signal-to-noise ratio are furnished. The stability and sensitivity analysis for topological derivative based detection functions for electromagnetic inclusions using far-field amplitude is not available in the literature, specially in the context of full Maxwell equations to the best of our knowledge.

First of all, the underlying medium containing an electromagnetic inhomogeneity is probed with an electric incident field and the far-field amplitude of the scattered electric field is recorded at the unit sphere. Then, a trial inhomogeneity is created at a search point in the background medium (without any inclusion) and is probed with the same incident field, rendering another far-field amplitude due to the trial inclusion. A discrepancy functional is then constructed and its minimizer is sought. The point relative to which the far-field amplitude of the trial inclusion minimizes the discrepancy is a potential candidate for the location of the true inclusion. Since