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LIMITED TOMOGRAPHY RECONSTRUCTION VIA TIGHT FRAME AND SIMULTANEOUS SINOGRAM EXTRAPOLATION*

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Abstract

X-ray computed tomography (CT) is one of widely used diagnostic tools for medical and dental tomographic imaging of the human body. However, the standard filtered back-projection reconstruction method requires the complete knowledge of the projection data. In the case of limited data, the inverse problem of CT becomes more ill-posed, which makes the reconstructed image deteriorated by the artifacts. In this paper, we consider two dimensional CT reconstruction using the projections truncated along the spatial direction in the Radon domain. Over the decades, the numerous results including the sparsity model based approach has enabled the reconstruction of the image inside the region of interest (ROI) from the limited knowledge of the data. However, unlike these existing methods, we try to reconstruct the entire CT image from the limited knowledge of the sinogram via the tight frame regularization and the simultaneous sinogram extrapolation. Our proposed model shows more promising numerical simulation results compared with the existing sparsity model based approach.

Mathematics subject classification: 65N20, 65N21, 94A08.
Key words: X-ray computed tomography, Limited tomography, Wavelet frame, Data driven tight frame, Bregmanized operator splitting algorithm, Sinogram extrapolation.

1. Introduction

X-ray computed tomography (CT) is a widely used diagnostic tool for medical and dental tomographic imaging of the human body. It provides tomographic images of the human body by assigning an X-ray attenuation coefficient to each pixel [1]. Let u denote the (unknown) image to be reconstructed. We further assume that u is supported in the unit ball $B(\mathbf{0},1)$ in $\mathbb{R}^2$. In the case of two dimensional parallel beam CT, the projection data (or sinogram) f for each $\varphi\in[0,2\pi)$ and $s\in\mathbb{R}$ is obtained via the following Radon transform [2]:

$$f(\varphi,s)=Pu(\varphi,s)=\int_{-\infty}^{\infty}u(s\boldsymbol{\theta}+t\boldsymbol{\theta}^{\perp})dt \tag{1.1}$$

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where $\boldsymbol{\theta} = (\cos\varphi, \sin\varphi)$ and $\boldsymbol{\theta}^{\perp} = (-\sin\varphi, \cos\varphi)$. Then the tomographic image u is reconstructed via the following Radon inversion formula [3,4]:

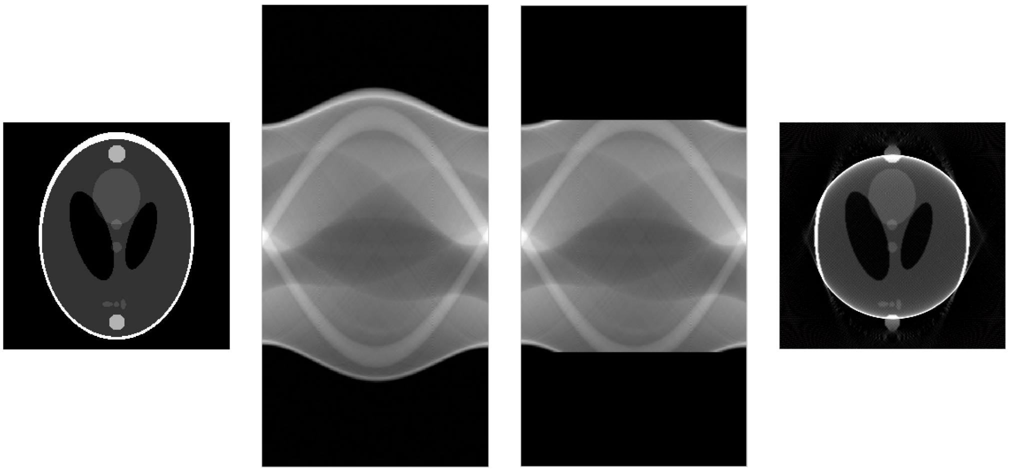
$$u(\boldsymbol{x}) = \frac{1}{2\pi^2}\int_0^{\pi}\int_{-\infty}^{\infty}\frac{1}{\boldsymbol{x}\cdot\boldsymbol{\theta} - s}\left[\frac{\partial}{\partial s}Pu(\varphi, s)\right] ds d\varphi \tag{1.2}$$

with $\boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2$.

However, the limitation of (1.2) lies in the fact that it requires so-called *complete data* [5–7], which means that the measured projection data should cover at least the range $(\varphi, s) \in [0, \pi) \times \mathbb{R}$. In the case of *limited data* where the sinogram $f(\varphi, s)$ is measured only on a subset of $[0, \pi) \times \mathbb{R}$ due to the reduced size of detector [8–10] and/or the reduced number of projections [11–13], the reconstruction is more ill-posed than the complete data case [5,14]. In particular, if the projection data is available only on $\Lambda_\mu \subseteq [0, \pi) \times \mathbb{R}$ given as

$$\Lambda_\mu = \Big\{(\varphi, s) \in [0, \pi) \times \mathbb{R} : |s| < \mu < 1\Big\}, \tag{1.3}$$

then there exists a nontrivial function g called the amgibuity of P [9], i.e. $Pg = 0$ in Λ_μ [9,15]. Since it has been proven that g is nonconstant in the region of interest (ROI) $B(\mathbf{0}, \mu)$ [4,9], the reconstructed image via (1.2) with the projection data f restricted on Λ_μ will be deteriorated by this ambiguity, as shown in Fig. 1.1.



(a) (b) (c) (d)

Fig. 1.1. The challenging issues in the limited tomography which is illustrated by (a) the original phantom image, (b) the full projection data covering $[0, \pi) \times \mathbb{R}$, (c) the limited data due to the detector size, and (d) the reconstructed image by (1.2) with the limited data. As we can see, the reconstructed image is corrupted by the ambiguity of P.

During the past decades, the development of CT theories has enabled the unique and stable reconstruction of CT image from the limited knowledge of the sinogram [8,10,15–20]. These existing reconstruction methods mainly focus on the reconstruction of the image inside $B(\mathbf{0}, \mu)$. However, since the sinogram restricted to Λ_μ may not necessarily agree with the projection of