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A PARAMETER-SELF-ADJUSTING LEVENBERG-MARQUARDT METHOD FOR SOLVING NONSMOOTH EQUATIONS*

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Abstract

A parameter-self-adjusting Levenberg-Marquardt method (PSA-LMM) is proposed for solving a nonlinear system of equations $F(x) = 0$, where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a semismooth mapping. At each iteration, the LM parameter μ_k is automatically adjusted based on the ratio between actual reduction and predicted reduction. The global convergence of PSA-LMM for solving semismooth equations is demonstrated. Under the BD-regular condition, we prove that PSA-LMM is locally superlinearly convergent for semismooth equations and locally quadratically convergent for strongly semismooth equations. Numerical results for solving nonlinear complementarity problems are presented.

Mathematics subject classification: 65K05, 90C30.
Key words: Levenberg-Marquardt method, Nonsmooth equations, Nonlinear complementarity problems.

1. Introduction

Suppose that a mapping $F : \mathbb{R}^n \to \mathbb{R}^n$ is locally Lipschitz but not necessarily continuously differentiable. We consider a nonlinear system of nonsmooth equations

$$F(x) = 0, \tag{1.1}$$

which arises from many important applications in optimization field. Pang and Qi [15] reviewed eight problems in the study of complementarity problems, variational inequality problems and optimization problems, which can be reformulated as systems of nonsmooth equations. From the early 1990s, a number of generalized Newton methods for solving nonsmooth equations were proposed and analyzed. For a comprehensive discussion on this topic, readers are referred to the monographs [4, 5] and references therein. In this paper, we will discuss the Levenberg-Marquardt-type methods for solving nonsmooth equations.

The Levenberg-Marquardt (LM) method proposed by [9, 11] is a classical and popular approach for solving nonlinear equations

$$H(x) = 0, \tag{1.2}$$

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where $H : \mathbb{R}^n \to \mathbb{R}^n$ is continuously differentiable. Define the merit function

$$h(x) := \frac{1}{2}\|H(x)\|^2,$$

then

$$\nabla h(x) = \mathcal{J}H(x)^T H(x).$$

For the $k-$th iterate point x^k, denote

$$\nabla h_k = \nabla h(x^k), \quad B_k = \mathcal{J}H(x^k)^T \mathcal{J}H(x^k).$$

At the $k-$th iteration, the LM method calculates the direction p^k by solving the following linear system of equations

$$[B_k + \mu_k I]p^k + \nabla h_k = 0, \tag{1.3}$$

where $\mu_k > 0$ is the LM parameter which is updated at each iteration. Note that, if B_k is nonsingular and $\mu_k = 0$, the LM direction p^k is reduced to the traditional Newton step. Thus, the LM method has global convergence and local quadratic convergence rate if the LM parameter μ_k is chosen suitably and the Jacobian B_k is nonsingular at the solution. However, the nonsingularity condition of the Jacobian is too strong. Yamashita and Fukushima [19] proved that, under the local error bound condition, the LM method maintains the quadratic convergence if the LM parameter is chosen as $\mu_k = \|F_k\|^2$. Fan and Yuan [7] chose $\mu_k = \|F_k\|^\delta$ with $\delta \in [1, 2]$ and proved that the LM method still achieves the quadratic convergence under the same conditions. We do not attempt to survey the literature on LM method for solving smooth equations, which is vast.

Based on the nonsmooth Newton-type method, Facchinei and Kanzow [3] studied a Levenberg-Marquardt-type algorithm with Armijo line search for solving the nonsmooth equation reformulation $\Phi(x) = 0$ of the nonlinear complementarity problems. At each iteration, select an element $H_k \in \partial\Phi(x^k)$ and find a solution d^k of the system

$$(H_k^T H_k + \mu_k I)d = -H_k^T \Phi(x^k).$$

If the BD-regular condition holds at a solution x^* and the LM parameter sequence $\{\mu_k\} \to 0$, the algorithm was shown to be superlinearly convergent. In the implementation of Levenberg-Marquardt-type algorithms for solving smooth equations, the choice of LM parameter μ_k is a critical issue. This paper aims to discuss the choice of LM parameter in nonsmooth case. It is well known that the Levenberg-Marquardt method can be viewed as a type of trust region method, see [13, 16] for instance. Inspired by a strategy of adjusting trust region radius, in this paper we will study a parameter-self-adjusting Levenberg-Marquardt method for solving nonsmooth equations, in which the parameter μ_k is automatically adjusted at each iteration.

The paper is organized as follows. In Section 2, we recall some concepts in nonsmooth analysis. In Section 3, we describe a nonsmooth parameter-self-adjusting Levenberg-Marquardt method and discuss its global convergence. Under mild conditions, the local convergence rate is analyzed in Section 4. Some preliminary numerical results for solving nonlinear complementarity problems are reported in Section 5.

2. Preliminaries

In this section, we review some definitions and results in nonsmooth analysis. We start by introducing the generalized differentials of a non-differentiable function, which is based on Rademacher's theorem.