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APPLICATION OF MFCAV RIEMANN SOLVER TO MAIRE'S CELL-CENTERED LAGRANGIAN METHOD*

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Abstract

In this paper, we apply arbitrary Riemann solvers, which may not satisfy the Maire's requirement, to the Maire's node-based Lagrangian scheme developed in [P. H. Maire et al., SIAM J. Sci. Comput, 29 (2007), 1781-1824]. In particular, we apply the so-called Multi-Fluid Channel on Averaged Volume (MFCAV) Riemann solver and a Riemann solver that adaptively combines the MFCAV solver with other more dissipative Riemann solvers to the Maire's scheme. It is noted that neither of the two solvers satisfies the Maire's requirement. Numerical experiments are presented to demonstrate that the application of the two Riemann solvers is successful.

Mathematics subject classification: 65N06, 65B99.
Key words: Maire's node-based Lagrangian scheme, Riemann solvers, Riemann invariants, weighted least squares procedure.

1. Introduction

Due to the significant advantages of capturing contact discontinuities and material interfaces sharply and automatically, Lagrangian methods have been rapidly developed in the past decades and are now widely used in many fields for compressible multi-material flow simulations, such as inertial confinement fusion (ICF) and astrophysics, see, e.g., [1, 8, 15, 18, 20], and the references therein.

Recently, P. H. Maire and his co-workers developed a node-based two-dimensional cell-centered Lagrangian scheme, see, e.g., [9-12]. The feature that distinguishes the Maire's scheme from other cell-centered Lagrangian schemes is that the conservations of momentum and total energy are satisfied at each node rather than in each cell, and in this way the node velocity is computed directly from the cell-centered quantities. In doing this, Riemann solvers located at the nodes are used across each cell edge emitting from the node, where the lacked degree of freedoms is supplied by introducing four pressures on each cell edge, two for each node on each side of each edge. In this way, the interface fluxes and the node velocity are computed in a compatible fashion, the geometric conservation law is thus satisfied, and the problems of artificial grid motion [6] and numerical sensitivity to the cell aspect ratio [5] that bother

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the cell-centered Lagrangian schemes are eased. Since then, a series of contributions to the development of the method have been presented and the method has been extended to the second-order scheme, Euler and ALE methods, solid dynamics, elastic-plastic flows, and so on, see e.g., [4, 10, 13, 17, 18].

In the Maire's scheme, the Riemann solvers to be used at nodes have to satisfy certain requirement, whose particular form is the acoustic Riemann solver, see (4.7) in [12] or (3.2) in §3 of this paper. We call it the Maire's requirement on the Riemann solvers in this paper. We believe, and our numerical experiment also confirms, see the discussion in §4, that this requirement is crucial for the feasibility of the scheme, with which the node velocity is computed from the normal velocities on the cell edges by solving a 2×2 linear system with positive definite coefficient matrix, see (4.13)-(4.14) in [12] or the discussion in §2 and §3 of this paper. We notice that this requirement on Riemann solvers has not been lifted in the later development of the method and the Riemann solvers used there are still restricted by this requirement, see the above mentioned papers. However, this requirement rejects many popularly used Riemann solvers in CFD.

The Multi-Fluid Channel on Averaged Volume (MFCAV) Riemann solver was proposed by Shui in 1980's [8], and some of its ideas can be traced back to von Neumann and Richtmyer in 1940-50's [22]. The solver is simple and can be easily applied to any fluids without request of information on the EOS's. Also it is much less dissipative, resulting sharper shock profiles and rarefaction corners in numerical simulations. The main drawback of the MFCAV Riemann solver is that it may produce spurious oscillations because of its less dissipation. To eliminate the spurious oscillations, the solver is often combined with other more dissipative Riemann solvers to form an adaptive Riemann solver(ADRS) in practical use. Because of its efficiency, the MFCAV Riemann solver and its ADRS's are still used in many application codes in Chinese CFD community, see e.g., [19, 20]. However, the MFCAV solver does not satisfy the Maire's requirement, neither do the ADRS's, see the discussion in §4. Therefore, they can not be straightforwardly applied to the Maire's scheme in the way as described in [12].

The main contribution of this paper is to develop a way for applying arbitrary Riemann solvers, even the solvers that do not satisfy the Maire's requirement, to the Maire's node-based Lagrangian scheme. In this way the feature of the Maire's scheme that the node velocity is computed from the normal velocities on the edges by solving a 2×2 linear system with positive definite coefficient matrix is preserved. This is achieved based on a re-formulation of the Maire's computation of the node velocity and half-pressures. In this paper, a starting one, we will only apply the MFCAV Riemann solver and an ADRS of it to the Maire's scheme.

We test the MFCAV Riemann solver because of the following two reasons: First, the MFCAV solver is still in our practical use because of its ability of handling discontinuities with great density and pressure jumps sharply. Second, the MFCAV solver is often used as a building block to construct more sophisticated and better Riemann solvers. Actually, the ADRS Riemann solver, see §4 for its description, is such an example and the numerical examples in §5 shows that it works better than the original acoustic Riemann solver in the scheme. For this reason, we need first to successfully apply the MFCAV solver to the Maire's scheme. In our future research, we are going to apply more different Riemann solvers, some of which may use the MFCAV solver as a building block, to the scheme. In this paper, we consider only the first-order semi-discretization as in [12], higher-order extensions will be our future researches. The rest of this paper is organized as follows.

In §2, we briefly recall the Maire's conservative cell-centered and node-based Lagrangian