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A NEW NONMONOTONE TRUST REGION ALGORITHM FOR SOLVING UNCONSTRAINED OPTIMIZATION PROBLEMS*

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Abstract

Based on the nonmonotone line search technique proposed by Gu and Mo (Appl. Math. Comput. 55, (2008) pp. 2158-2172), a new nonmonotone trust region algorithm is proposed for solving unconstrained optimization problems in this paper. The new algorithm is developed by resetting the ratio ρ_k for evaluating the trial step d_k whenever acceptable. The global and superlinear convergence of the algorithm are proved under suitable conditions. Numerical results show that the new algorithm is effective for solving unconstrained optimization problems.

Mathematics subject classification: 90C30.
Key words: Unconstrained optimization problems; Nonmonotone trust region method; Global convergence; Superlinear convergence.

1. Introduction

We consider the following unconstrained optimization problem

$$\min_{x\in\Re^n} f(x), \tag{1.1}$$

where $f:\Re^n\to\Re$ is a continuously differentiable function.

Line search method and trust region method are two principal methods for solving unconstrained optimization problem (1.1). Line search method produce a sequence $x_0, x_1, \cdots$, where x_{k+1} is generated from the current approximate solution x_k, the specific direction d_k and a stepsize $\alpha_k>0$ by the rule $x_{k+1}=x_k+\alpha_k d_k$. On the other hand, the trust region methods obtain a trial step d_k by solving the following quadric program subproblem:

$$\begin{aligned} \min_{d\in\Re^n} \quad & \phi_k(d)=g_k^Td+\frac{1}{2}d^TB_kd, \\ s.t. \quad & \|d\|\le\Delta_k, \end{aligned} \tag{1.2}$$

where $g_k=\nabla f(x_k), B_k\in\Re^{n\times n}$ is a symmetric matrix which is either the exact Hessian matrix of f at x_k or an approximation for it, $\Delta_k>0$ is the trust region radius and $\|\cdot\|$ denotes the Euclidean norm. The traditional trust region methods evaluate the trial step d_k by the ratio

$$\rho_k=\frac{f(x_k)-f(x_k+d_k)}{\phi_k(0)-\phi_k(d_k)}. \tag{1.3}$$

The trial step d_k is accepted whenever ρ_k is greater than a positive constant μ, then we can set the new point $x_{k+1}=x_k+d_k$ and enlarge the trust region radius. Otherwise, the traditional

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trust region methods resolve the subproblem (1.2) by reducing the trust region radius until an acceptable step is found. Solving the region subproblems may lead us to solve one or more quadric program problems and increase the total cost of computation for large scale problems. Compared with line search techniques, new trust region methods have a strong convergence property, its computational cost is much lower than the traditional trust region methods (e.g. [1-3]). Some theoretical and numerical results of these trust region methods with line search are also interesting.

However, all these methods enforce monotonically decreasing of the objective function values at each iteration may slow the convergence rate in the minimization process, see [4,8]. In order to overcome the shortcomings, the earliest nonmonotone line search framework was developed by Grippo, Lampariello and Lucidi in [5] for Newton's method, in which their approach was: parameters $\lambda_1, \lambda_2, \sigma$ and β are introduced, where $0 < \lambda_1 < \lambda_2$, $\beta, \sigma \in (0,1)$ and $\alpha_k = \bar{\alpha}_k \sigma^{h_k}$, where $\bar{\alpha}_k \in (\lambda_1, \lambda_2)$ is the trial step and h_k is the smallest nonnegative integer such that

$$f(x_k + d_k) \leq \max_{0 \leq j \leq m_k} f(x_{k-j}) + \beta \alpha_k \nabla f(x_k)^T d_k, \tag{1.4}$$

the memory m_k is a nondecreasing integer by setting

$$m_k = \begin{cases} 0, & k = 0, \\ 0 < m_k \leq \min\{m_{k-1} + 1, M\}, & k > 0, \end{cases}$$

where M is a prefixed nonnegative integer.

From then on, researches in nonlinear optimization area have paid great attentions to it, see [7,8,10-14]. Deng et al. [4] made some changes in the common ratio (1.3) by resetting the rule as follows:

$$\hat{\rho}_k = \frac{\max\limits_{0 \leq j \leq m_k} f(x_{k-j}) - f(x_k + d_k)}{\phi_k(0) - \phi_k(d_k)}. \tag{1.5}$$

The ratio (1.5) which assesses the agreement between the quadratic model and the objective function in trust region methods is the most common nonmonotone ratio, some researchers showed that the nonmonotone method can improve both the possibility of finding the global optimum and the rate of convergence when a monotone scheme is forced to creep along the bottom of a narrow curved valley (see [4,9,17]).

Although the nonmonotone technique (1.4) has many advantages, Zhang and Hager [18] proposed a new nonmonotone line search algorithm, which had the same general form as the scheme of Grippo et al. [5], except that their "max" was replaced by an average of function values. The nonmonotone line search found a step length β to satisfy the following inequality:

$$f(x_k + \beta d_k) \leq C_k + \delta \beta \nabla f(x_k)^T d_k, \tag{1.6}$$

where

$$C_k = \begin{cases} f(x_k), & k = 0, \\ \big(\eta_{k-1} Q_{k-1} C_{k-1} + f(x_k)\big)/Q_k, & k \geq 1, \end{cases} \tag{1.7}$$

$$Q_k = \begin{cases} 1, & k = 0, \\ \eta_{k-1} Q_{k-1} + 1, & k \geq 1, \end{cases} \tag{1.8}$$

$\eta_{k-1} \in [\eta_{\min}, \eta_{\max}]$, where $\eta_{\min} \in [0,1)$ and $\eta_{\max} \in [\eta_{\min}, 1]$ are two chosen parameters. Numerical results showed that the new nonmonotone can improve the efficiency of the nonmonotone trust region methods.