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EVALUATION OF SINGULAR AND NEARLY SINGULAR INTEGRALS IN THE BEM WITH EXACT GEOMETRICAL REPRESENTATION*

Yaoming Zhang
Institute of Applied Mathematics, Shandong University of Technology, Zibo 255049, China
State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China
Email: zymfc@163.com
Wenzhen Qu   Yan Gu
Center for Numerical Simulation software in Engineering and Sciences, Department of Engineering Mechanics, Hohai University, Nanjing 210098, China
Email: qwzxxoo7@163.com   guyan1973@163.com

Abstract

The geometries of many problems of practical interest are created from circular or elliptic arcs. Arc boundary elements can represent these boundaries exactly, and consequently, errors caused by representing such geometries using polynomial shape functions can be removed. To fully utilize the geometry of circular boundary, the non-singular boundary integral equations (BIEs) and a general nonlinear transformation technique available for arc elements are introduced to remove or damp out the singular or nearly singular properties of the integral kernels. Several benchmark 2D elastostatic problems demonstrate that the present algorithm can effectively handle singular and nearly singular integrals occurring in the boundary element method (BEM) for boundary layer effect and thin-walled structural problems. Owing to the employment of exact geometrical representation, only a small number of elements need to be divided along the boundary and high accuracy can be achieved without increasing other more computational efforts.

Mathematics subject classification: 68Q05.
Key words: BEM, Singular integrals, Nearly singular integrals, Boundary layer effect, Thin walled structures, Exact geometrical representation.

1. Introduction

The BEM is a powerful and efficient computational method if boundary integrals can be evaluated accurately. The main advantage of the BEM resulting from the reduction of the dimension of the boundary value problem is well-known. However, it is popular as well that the standard BEM formulations include singular and nearly singular integrals, and thus the integrations should be performed very carefully. In the past decades, many direct and indirect algorithms for singular integral have been developed and used successfully [1-11]. The nearly singular integrals, however, need to be further studied, although great progresses have been achieved for each of the existing methods. Studies show that the conventional boundary element method (CBEM) using the standard Gaussian quadrature fails to yield reliable results for nearly singular integrals. The major reason for this failure is that the kernels' integration

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presents various orders of near singularities since the integrand oscillates very fiercely within the integration interval. Therefore, although nearly singular integrals are not singular in the sense of mathematics, it can not be calculated accurately by the standard Gaussian quadrature.

Nearly singular integrals usually occur for the thin-body problem when the thickness of the considered domain is small, or for the case where the physical quantity is calculated at a domain point which is very close to the boundary, or for the case where the mesh contains a large element and a small element adjacent to each other. The usual approach to achieving high accuracy is to use the subdivision method which is done to increase the number of subdivisions as the source point gets close to the element where the integral is taken. However, this method requires too much preprocessing and CPU time, especially for solving thin-body problems.

In the past decades, tremendous effort was devoted to derive convenient integral forms or sophisticated computational techniques for calculating the nearly singular integrals. These proposed methods can be divided on the whole into two categories: "indirect algorithms" and "irect algorithms". The indirect algorithms [4,9-11], which benefit from the regularization ideas and techniques for the singular integrals, are mainly to calculate indirectly or avoid calculating the nearly singular integrals by establishing new regularized boundary integral equations (BIEs). However, the accuracy of their calculated results is not very satisfactory. The direct algorithms are calculating the nearly singular integrals directly. They usually include, but are not limited to, interval subdivision method [12-13], special Gaussian quadrature method [14-15], the exact integration method [16-18], and nonlinear transformation method [19-23]. Although great progresses have been achieved for each of the above methods, it should be pointed out that the geometry of the boundary element is often depicted by using linear shape functions when nearly singular integrals need to be calculated [22]. However, most engineering processes occur mostly in complex geometrical domains, and obviously, higher order geometry elements are expected to be more accurate to solve such practical problems [1-4]. Recently, two regularized algorithms suitable for calculating the nearly singular integrals occurring on the high-order geometry elements was proposed by the authors of this paper [18,23].

It is well known that the accuracy is a vital factor in a successful calculation, together with minimum computer storage and CPU time. There are several sources of inaccuracy in the boundary element method. They are the use of polynomial shape functions to represent the boundary geometries, also to represent the variations of the physical variables over the boundary, the numerical integrations using Gaussian quadrature, and the rounding off errors during the evaluations.

The geometries of many problems of practical interest are created from circular or elliptic arcs. Arc boundary elements can represent circular and elliptic boundaries exactly, and consequently, errors caused by representing such geometries using polynomial shape functions can be removed by using exact geometrical representations. Therefore, the exact geometrical representation is expected to give more accurate results than lower-order or even high-order boundary element analysis.

To fully utilize the geometry of circular boundary, the approach used in this paper is exact geometrical representation for circular and elliptic boundaries. Both singular and nearly singular integrals are reconstructed and calculated under such geometrical representation. To verify the method developed in this paper, both boundary layer effect and thin body problems are considered. For boundary layer effect, the stresses at the interior points very close to boundary are evaluated. For thin body problems, very promising results are obtained when the thickness-to-length ratios is in the orders from 10^{-1} to 10^{-9}, which is sufficient for mod-