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AN OPTICAL FLOW APPROACH TO ANALYZING SPECIES DENSITY DYNAMICS AND TRANSPORT*

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Abstract

Classical optical flow techniques were developed for computing virtual motion fields between two images of the same scene, assuming conservation of intensity and a smoothness of the flow field. If these assumptions are dropped, such techniques can be adapted to compute apparent flows between time snapshots of data that do not come from images, even if these flows are turbulent and divergent, as in the case of flows representing complex spatiotemporal dynamics. While imaging methods have been used to analyze dynamics in experimental applications, they are only beginning to be applied to dynamics computations in settings outside the laboratory, for example in the analysis of species population dynamics from satellite data. In this work we present a variational optical flow approach based on the continuity equation and total variation regularization for computing the flow fields between population densities generated from a two-species predator-prey model for phyto- and zooplankton interactions. Given the time-varying vector fields produced from the optical flow, computational methods from dynamical systems can be employed to study pseudo-barriers present in the species interaction. This method allows to measure the mixing of the species, as well as the transport of the populations throughout the domain.

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Key words: Optical flow, Finite-time Lyapunov exponent, Mass transport, Data-driven dynamical systems.

1. Introduction

Variational optical flow methods are used to compute dense vector fields describing apparent motions between two adjacent images. The original optical flow algorithm, described by Horn and Schunck in [20], was based on the assumptions of local conservation of intensity and of the smoothness of the flow field. These assumptions are appropriate when the flow between the images is divergence-free, which is reasonable to assume for many rigid-body motions, particularly when the surfaces being imaged are not highly reflectant. The mathematical principle of optical flow, however, can serve as motivation and be adapted to compute flow fields between two different time snapshots of other kinds of data, even data that do not represent intensities *per se*.

One of the primary reasons to be interested in computing flow fields from non-image data is that in recent years new methods have been developed for analysis of dynamical systems from

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the vector fields that describe the mass transport of the system. One such technique is the use of finite-time Lyapunov exponents (FTLE) [15] for discerning Lagrangian Coherent Structures (LCS) [16] in a dynamical system. The main idea behind the LCS computation is that it allows the detection and analysis of transport boundaries, describing a kind of time-varying region segmentation based on the dynamics of the system. That is, the coherent structures describe which regions of mass or density will flow together or mix in a system and which will flow away from each other. In most such analyses, the flow fields are derived directly from a system model using analytical techniques. In the absence of a model – or if the model used does not sufficiently capture the true dynamics – it is necessary to generate the governing flow field directly from measurements. This has been done using imaging techniques, for example, in particle image velocimetry (PIV) experiments [6, 24], but not for many applications outside of the laboratory. In particular, there is little work performing dynamics analysis directly from remotely sensed data in a model-free environment. The ability to perform such analysis would be particularly useful, since many dynamical systems are based on measured data with only approximate supporting *a priori* models or without any supporting models at all. In this work we adapt optical flow techniques to this end.

In order to demonstrate dynamics analysis using optical flow, we focus on a two-species predator-prey system of two partial differential equations in two unknown functions that model how the spatial densities of two species of plankton evolve in time. Given the density of one of the species at a particular time, optical flow techniques can be used to compute the virtual flow of the population density to the next discrete time step. In such a case, the flows of the densities are highly divergent and also have dynamic behavior not observed in classical optical flow applications, so it is necessary to adapt the current optical flow algorithms to allow for divergent flows and to capture dynamic structures such as vortices. The resulting flow fields are then used to compute the FTLE field for the flow as well as the density transport boundaries and pseudo-boundaries.

In this work, we demonstrate dynamics analysis from optical flow by generating synthetic data from a known model for plankton interactions. The model is used only for generating the data, and the optical flow and dynamics analysis are performed directly on the resulting densities, which are treated as measured data. This sample system is used to demonstrate the technique and to show that, if the system data were measured, for example by hyperspectral sensors, the dynamics could be computed directly from the measured intensities. Thus adapting methods such as optical flow from imaging science and computer vision is an ideal approach for analyzing the system dynamics, since such techniques can be generalized and applied directly to measured data. The primary advances of this work are the adaptation of optical flow to dynamical systems analysis for systems that are driven by data measured in an uncontrolled environment and the development of a computational approach suited to this application. Thus dynamics analysis which has previously been available only for systems with accurate models or in controlled laboratory environments can now be performed on model-free systems that are driven by data measured in uncontrolled environments.

In Section 2 the theoretical formulation of the variational optical flow approach is developed. The background, theory, and computational details of the LCS methodology are outlined in Section 3. Section 4 highlights the two-species predator-prey system for modeling phyto- and zooplankton population densities over time. The optical flow is computed directly from the generated data – and not from the model itself – serving as a test case for computing flow fields in the absence of models. The apparent flow fields are thus computed directly using optical