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A SMOOTHING TRUST REGION METHOD FOR NCPS BASED ON THE SMOOTHING GENERALIZED FISCHER-BURMEISTER FUNCTION*

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Abstract

Based on a reformulation of the complementarity problem as a system of nonsmooth equations by using the generalized Fischer-Burmeister function, a smoothing trust region algorithm with line search is proposed for solving general (not necessarily monotone) nonlinear complementarity problems. Global convergence and, under a nonsingularity assumption, local Q-superlinear/Q-quadratic convergence of the algorithm are established. In particular, it is proved that a unit step size is always accepted after a finite number of iterations. Numerical results also confirm the good theoretical properties of our approach.

Mathematics subject classification: 90C33, 90C30.
Key words: Nonlinear complementarity problem, Smoothing method, Trust region method, Global convergence, Local superlinear convergence.

1. Introduction

Let $F : R^n \to R^n$ be continuously differentiable. The nonlinear complementarity problem, denoted by NCP(F), is to find a vector $x \in R^n$ such that

$$x \geq 0, \quad F(x) \geq 0, \quad \langle x, F(x)\rangle = 0. \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ is the Euclidean inner product. If F is an affine function of x, then NCP(F) reduces to a linear complementarity problem (LCP). The NCP is theoretically and practically useful, and has been used to study and formulate various equilibrium problems in economics and engineering, such as Nash equilibrium problems, traffic equilibrium problems, contact mechanics problems and so on [1, 2].

There have been many methods proposed for the solution of NCP(F). Among them, one of the most popular and powerful approaches that has been studied intensively recently is to reformulate NCP(F) as a system of nonlinear equations [10], as an unconstrained minimization problem using suitable merit functions [7], or as a parametric problem. Here we concentrate on the equation-based method, where the NCP(F) can be written equivalently as

$$\Phi(x) = 0 \tag{1.2}$$

for a suitable equation operator $\Phi : R^n \to R^n$. Recently, for solving complementarity problems, various equivalent equation-based reformulations have been proposed and seem attractive. For

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more details of these reformulations, please see [6,10,12,14,15,19]. Generally the operator Φ is locally Lipschitz continuous but not differentiable, so that we cannot apply the classical Newton method directly to solve Eq. (1.2). Nevertheless, recent research shows that one can still design globally and locally fast convergent algorithms for the solution of Eq. (1.2). Most of these methods are classified by Kanzow and Pieper [20] into three categories: namely, nonsmooth Newton methods, Jacobian smoothing methods and smoothing (Newton) methods. We refer the interested reader to [20] and references therein.

In this paper, we aim to develop a trust region-type method for solving nonsmooth equations (1.2). Trust region methods for solving nonsmooth equations have been studied in [3, 9, 11]. The algorithm proposed in [3] was devoted to solving a semismooth equation reformulation for generalized complementarity problems by adopting the squared natural residual of the semismooth equations as a merit function. Actually, it used a trust region strategy for solving the following unconstrained minimization problem

$$\min_{x\in R^n} \ \widetilde{\Phi}(x),$$

where its merit function $\widetilde{\Phi}(x) := \|\Phi(x)\|_2^2/2$. In their method, the trust region subproblem was the following minimization problem

$$\begin{cases} \min & \nabla\widetilde{\Phi}(x^k)^T d + \dfrac{1}{2} d^T V_k^T V_k d \\ \text{s.t.} & \|d\| \le \Delta_k, \end{cases}$$

where Δ_k was the trust region radius and V_k was an arbitrary element in the Clarke's [13] generalized Jacobian of Φ at x^k. Global convergence and, under a nonsingularity assumption, local Q-superlinear (or Q-quadratic) convergence of this trust region method were established.

Inspired by Jiang *et al.*'s work [3], we develop a smoothing trust region method for solving the NCPs. The method is based on the recently presented smoothing technique which is to construct a smoothing approximation function $\Phi_\mu : R^n \to R^n$ of Φ such that for any $\mu > 0$, Φ_μ is continuously differentiable and

$$\|\Phi(x) - \Phi_\mu(x)\| \to 0, \qquad \text{as } \mu \downarrow 0 \text{ for all } x \in R^n.$$

The parameter μ is called smoothing parameter. Based on the smoothing function Φ_μ, we define a merit function

$$\theta_\mu(x) := \frac{1}{2}\|\Phi_\mu(x)\|^2 \tag{1.3}$$

and propose a trust region method where, at each iterate point x^k, the trial step d^k is obtained by solving the following trust region subproblem,

$$\begin{cases} \min & \Theta(d) := \dfrac{1}{2}\|\Phi_{\mu_k}(x^k) + J\Phi_{\mu_k}(x^k)d\|^2 \\ \text{s.t.} & \|d\| \le \Delta_k. \end{cases} \tag{1.4}$$

And define the actual reduction and the predicted reduction as follows:

$$\begin{aligned} Ared_k &:= \theta_{\mu_k}(x^k) - \theta_{\mu_k}(x^k + d^k), \\ Pred_k &:= \Theta(0) - \Theta(d^k). \end{aligned}$$