

LOCAL CONVERGENCE OF AN EM-LIKE IMAGE RECONSTRUCTION METHOD FOR DIFFUSE OPTICAL TOMOGRAPHY*

Caifang Wang

College of Art and Sciences, Shanghai Maritime University, Shanghai 200135, China

LMAM, School of Mathematical Sciences, Peking University, Beijing 100871, China

Email: wangcfg@gmail.com

Tie Zhou

LMAM, School of Mathematical Sciences, Peking University, Beijing 100871, China

Email: tzhou@math.pku.edu.cn

Abstract

In this paper, an EM-like image reconstruction iterative formula specifically developed for stable external sources is rewritten as a map towards a fixed point iteration. Local convergence of the image reconstruction method is then proved. Finally a three-dimensional numerical image reconstruction example is presented.

Mathematics subject classification: 78A70, 78M50, 93B15.

Key words: Diffuse optical tomography, Image reconstruction, Fixed-point iteration, Local convergence.

1. Introduction

Diffuse optical tomography (DOT) is an optical imaging modality, which provides the spatial distribution of the optical parameters inside a random media [1]. This nondestructive technique has advantage of directly measuring the physiologically relevant tissue and blood oxygenation, and is now widely used in breast cancer diagnostics [2, 3], joint imaging [4] and blood oximetry in human muscle and brain tissues [5, 6].

In DOT, the near-infrared external sources are used to delivery the light signals. The intensity and path-length distributions of the exiting photons provide information about the optical properties of the transilluminated tissue by means of a physical models of the light migration. The propagation of light in highly scattering media, such as biological tissue may be described by the radiative transfer equation (RTE) [7]. When the medium is predominantly scattering rather than absorption, the diffusion approximation (DA) is a good approach to the RTE, away from sources and boundaries and it has been widely used in DOT [7, 8].

Mathematically, the image reconstruction of DOT is an inverse problem solving the absorption and diffusion coefficient from the boundary measurements. Various reconstruction methods based on DA model have been established. The analytical methods with different boundary conditions are studied in a series of papers [9–11] and they can reconstruct optical parameters only in simple region cases. The iterative optimization based reconstruction methods [12] are used widely since they can deal with optical parameters in complex regions. In

* Received August 2, 2009 / Revised version received October 8, 2009 / Accepted January 13, 2010 /
Published online September 20, 2010 /

these methods, the problem is regarded as the optimization of an objective function representing the sum-squared difference of the data to the model, plus additional regularization terms representing prior knowledge. An EM-like reconstruction method for stationary sources DOT is proposed, see, e.g., [13]. In this method, the boundary measurements are assumed to have independent and identical Poisson distributions. The problem is regarded as the optimization of a log-likelihood function with nonnegative constrain of optical parameters.

In this paper, we investigate the convergence of the EM-like image reconstruction which is specifically developed for stable external sources condition. The rest of the paper is organized as follows. In Section 2, we review the photon migration model and introduce the stationary sources DOT forward and inverse problems. In Section 3 we derive the EM-like image reconstruction algorithm. In Section 4, the local convergence of this algorithm is proved. In Section 5, a 3-D numerical example is presented.

2. DOT Forward and Inverse Problems

Let $\Omega \subset \mathbb{R}^3$ be a domain that contains the tissue to be imaged, bounded by surface $\Gamma = \partial\Omega$. Let $\mu_a(x)$ and $\mu_s(x)$ be the absorption and scattering coefficients of the tissue, respectively. Denote $D(x)$ as the diffusion coefficient which is expressed as

$$D(x) = \frac{1}{3(\mu_a(x) + \mu'_s(x))}, \quad (2.1)$$

where $\mu'_s = (1 - \bar{\eta})\mu_s$ is the reduced scattering coefficient of the media and $\bar{\eta}$ is an anisotropy factor ($0 \leq \bar{\eta} \leq 1$).

We consider the cases in which the tissue to be imaged is illuminated by multiple stationary external sources. Denote S as the number of external sources. Under each irradiation, the measurement can be collected on part of the surface $\Gamma_i \subset \Gamma$, $i = 1, \dots, S$.

Individual photons from the stationary external source migrate through the tissue and undergo many scattering events or absorption events according to the local values of the tissue's optical parameters. Each photon may either have negligible contribution, or escape from the surface $\partial\Omega$, thus contributing to the boundary measurements. Under each irradiation, the macroscopic phenomena of photons can be described with a steady state diffusion equation (DA model)

$$-\nabla \cdot (D(x)\nabla u_i(x)) + \mu_a(x)u_i(x) = 0, \quad x \in \Omega, \quad (2.2)$$

where $u_i(x)$ is the isotropic photon density inside Ω for $i = 1, \dots, S$. The external source information is contained in the boundary condition

$$u_i(x) + 2AD(x)\frac{\partial u_i}{\partial \nu}(x) = g_i^-(x), \quad x \in \Gamma, \quad (2.3)$$

where $g_i^-(x)$ is the total inward flux, A is a parameter that describes the mismatch between the refractive index within Ω and the refractive index in the surrounding medium [14, 15], ν is the exterior normal. The steady-state attenuation measurements are collected on part of the surface $\Gamma_i \subset \Gamma$, $i = 1, \dots, S$, and can be defined as the outward flux

$$g_i(x) = -D\frac{\partial u_i}{\partial \nu}(x), \quad x \in \Gamma_i, \quad i = 1, \dots, S. \quad (2.4)$$