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TWO-GRID DISCRETIZATION SCHEMES OF THE NONCONFORMING FEM FOR EIGENVALUE PROBLEMS *

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Abstract

This paper extends the two-grid discretization scheme of the conforming finite elements proposed by Xu and Zhou (Math. Comput., 70 (2001), pp.17-25) to the nonconforming finite elements for eigenvalue problems. In particular, two two-grid discretization schemes based on Rayleigh quotient technique are proposed. By using these new schemes, the solution of an eigenvalue problem on a fine mesh is reduced to that on a much coarser mesh together with the solution of a linear algebraic system on the fine mesh. The resulting solution still maintains an asymptotically optimal accuracy. Comparing with the two-grid discretization scheme of the conforming finite elements, the main advantages of our new schemes are twofold when the mesh size is small enough. First, the lower bounds of the exact eigenvalues in our two-grid discretization schemes can be obtained. Second, the first eigenvalue given by the new schemes has much better accuracy than that obtained by solving the eigenvalue problems on the fine mesh directly.

Mathematics subject classification: 65N25, 65N30.
Key words: Nonconforming finite elements, Rayleigh quotient, Two-grid schemes, The lower bounds of eigenvalue, High accuracy.

1. Introduction

Xu [10–12] first proposed two-grid discretization methods for nonsymmetric and nonlinear elliptic problems. Later, Xu and Zhou [13] proposed a two-grid discretization scheme of conforming finite elements for eigenvalue problems. In [14], Xu and Zhou proposed some local and parallel finite element algorithms based on [13]. Yang [15] extended the method in [13] to the Wilson nonconforming element and demonstrated by numerical experiments that the first eigenvalue given by the two-grid discretization scheme approximates the exact eigenvalue from below and has much better accuracy than that obtained by solving the eigenvalue problem on a fine mesh directly.

In this paper we will discuss two-grid discretization schemes of the nonconforming finite elements for any n-dimensional eigenvalue problems. We propose a new two-grid discretization scheme (see Scheme 1) and extend the scheme in [13, 15] (see Scheme 2). Using these two new schemes, the solution of an eigenvalue problem on a fine mesh is reduced to the solution of an eigenvalue problem on a much coarser mesh and the solution of a linear algebraic system on the fine mesh and the resulting solution still maintains an asymptotically optimal accuracy. Comparing with the two-grid discretization scheme of the conforming finite elements (see [13]), the main advantages of our new schemes are twofold when the mesh size is small enough. First, the lower bounds of the exact eigenvalues in our two two-grid discretization schemes can be

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obtained. Second, the first eigenvalue given by these new schemes has much better accuracy than that obtained by solving the eigenvalue problems on fine mesh directly.

Finding the lower bounds of eigenvalues by using the nonconforming elements has attracted many attentions in the past. In 1967, Zienkiewicz etc. [19] discovered that the nonconforming Morley element approximates eigenvalues from below. As for the vibration plate problems, Rannacher [6] provided numerical results in 1979, which indicated that the nonconforming Morley and Adini element can be used to obtain lower bounds of eigenvalues. Yang [16] proved that on a rectangular domain, the Adini element approximates the exact eigenvalues from below. For Laplace operator eigenvalue problems, Armentano and Duran [1] proved that piecewise linear nonconforming Crouzeix-Raviart element approximates the exact eigenvalues from below, Lin and Lin [5] proved that the nonconforming EQ_1^{rot} element approximates the exact eigenvalues from below, and Zhang et al. [18] proved that the nonconforming Wilson element approximates the exact eigenvalues from below. In this paper, we will show that the proposed two-grid discretization schemes maintain the above properties of approximation from below.

By the minimum-maximum principle we can conclude that the first eigenvalue given by the two-grid discretization scheme in [13] has much lower accuracy than that obtained by solving the eigenvalue problems on the fine mesh directly for conforming finite elements. However, to our surprise, it is exactly opposite for two-grid discretization scheme of most nonconforming finite elements. In particular, the two-grid discretization schemes of nonconforming finite elements are very efficient for eigenvalue problems.

The rest of the paper is organized as follows. In Section 2, we shall describe some notation and properties of the nonconforming finite element approximation for eigenvalue problems. In Section 3, we propose two two-grid discretization schemes of the nonconforming finite elements for eigenvalue problems and discuss approximation properties of the schemes. In Section 4, we apply the results in Section 3 to several representative nonconforming finite elements such as Wilson, Crouzeix-Raviart and Adini nonconforming elements.

2. Preliminaries

Let Ω be a bounded open connected subset of R^n with a Lipschitz-continuous boundary. Let V be a mth-order Sobolev space over Ω with inner product $(\cdot,\cdot)_V$ and norm $\|\cdot\|_V$ (m=1, 2), and let W be a sth-order Sobolev space over Ω with inner product $(\cdot,\cdot)_W$ and norm $\|\cdot\|_W$ $(0 \le s < m), V \subset W$ with a compact imbedding.

Suppose that $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$ are symmetric and continuous bilinear forms on $V \times V$ and $W \times W$, respectively, which satisfy

$$\begin{aligned}
&\mid a(u,v) \mid \le M_1\|u\|_V\|v\|_V, && \forall u,v \in V,\\
&a(u,u) \ge \alpha_1\|u\|_V^2, && \forall u \in V,\\
&\mid b(u,v) \mid \le M_2\|u\|_W\|v\|_W, && \forall u,v \in W,\\
&b(u,u) \ge \alpha_2\|u\|_W^2, && \forall u \in W.
\end{aligned}$$

Define $\|\cdot\|_b = b(\cdot,\cdot)^{\frac{1}{2}}$. Noting that $\|\cdot\|_b$ and $\|\cdot\|_W$ are two equivalent norms on W, we shall use $b(\cdot,\cdot)$ and $\|\cdot\|_b$ as the inner product and norm on W, respectively.

Consider the $2m$th-order elliptic differential operator eigenvalue problems: Find $(\lambda,u) \in R \times V, \|u\|_b = 1$ satisfying

$$a(u,v) = \lambda b(u,v), \quad \forall v \in V. \tag{2.1}$$