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REFLECTION/TRANSMISSION CHARACTERISTICS OF A DISCONTINUOUS GALERKIN METHOD FOR MAXWELL'S EQUATIONS IN DISPERSIVE INHOMOGENEOUS MEDIA*

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Dedicated to Professor Junzhi Cui on the occasion of his 70th birthday

Abstract

In this paper, we analyze the transmission and reflection properties of a high order discontinuous Galerkin method for dispersive Maxwell's equations, originally proposed by Lu et al. [J. Comput. Phys. **200** (2004), pp. 549-580]. We study the reflection and transmission properties of the numerical method for up to second-order polynomial elements for one- and two-dimensional Maxwell's equations with rectangular meshes. High order accuracy has been shown for reflection and transmission coefficients near material interfaces.

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Key words: Discontinuous Galerkin Method, Reflection, Transmission.

1. Introduction

Wave propagation in inhomogeneous and dispersive media can be found in many scientific and engineering applications such as evanescent waves in surface plasmons, and ground penetrating radar detection, and optical devices. In those situations, the need for accurate numerical modeling calls for continuing advances in the development of high order numerical algorithms. An important desirable feature is the capability of the numerical methods in predicting accurately the reflection and transmission of waves across material interfaces. For electromagnetic scattering in a dispersive media, the frequency dependent constitutive relation between displacement field D and the electric field E entails a time domain relationship via a time convolution. In [1, 2], the Auxilary Differential Equation (ADE) method is proposed to address this issue in the framework of discontinuous Galerkin methods for dispersive Maxwell's equations, and various applications of the resulting dispersive discontinuous Galerkin method have been conducted for the modeling of ground penetrating radar [3], resonant microcavity waveguide [4], and plasmon coupling of nanowires [2].

In this paper, we will elaborate the transmission and reflection properties of the above mentioned discontinuous Galerkin method for Maxwell's equation in a dispersive media with material interfaces. The reflection/transmssion near a material interface has a great effect on the quality of the simulation of wave propagation in a dispersive inhomogeneous media. Many physical phenomena involves waves near material interfaces, such as evanescent plasmon waves

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near dielectric and metals, which decay exponentially away from interfaces, and diffractive optics with gratings, to just list a few. Numerical modeling of the waves near material interfaces requires high fidelity in replicating the reflection and transmission of waves near material interfaces and the resolution power of approximating exponentially decaying field distributions.

In this paper, we are mainly concerned with the reflection/transmission property of the discontinuous Galerkin method proposed in [1,2] for the dispersive Maxwell's equations. Such an analysis is important for selecting numerical algorithms for studying electromagnetic waves near material interface and in dispersive media such as soil and metals and even in artificial dispersive media of PML for truncating computational domain [5]. The rest of the paper is organized as follows. In Section 2, we give an introduction of dispersive Maxwell's equations. In Section 3, the analysis of reflection/transmission properties of discontinuous Galerkin method for 1-D Maxwell's equations is given. In Section 4, we investigate the reflection/transmission properties of discontinuous Galerkin method for 2-D Maxwell's equations with rectangular meshes. Section 5 contains the conclusion.

2. Dispersive Maxwell's Equations

Maxwell's equations are fundamental equations of electromagnetism, which are of the form

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}, \tag{2.1}$$

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{j}, \tag{2.2}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{2.3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.4}$$

where $\mathbf{E}$ is the electric field and $\mathbf{B}$ the magnetic induction, $\mathbf{D}$ electric displacement, $\mathbf{H}$ the magnetic field, ρ the free charge density and $\mathbf{j}$ the free current density.

Eqs. (2.1)-(2.4) are not closed in themselves; they must be supplemented with constitutive relations

$$\mathbf{D} = \epsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}, \quad \mathbf{j} = \sigma\mathbf{E}, \tag{2.5}$$

where ϵ is the permittivity, μ is the permeability, and σ is the conductivity. For dispersive media, ϵ in (2.5) is a function of the frequency ω.

For a lossy and dispersive media, a typical single-pole Drude medium [6] has a relative frequency dependent electric permittivity as

$$\hat{\epsilon}_r(\omega) = \hat{\epsilon}_{r,\infty} - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{2.6}$$

where ω_p is the plasma frequency, γ is the damping constant, $\hat{\epsilon}_{r,\infty}$ is the relative electric permittivity at infinite frequency. Fourier transforms are needed to get an expression in time domain.

In [1,4], a discontinuous Galerkin method employs ADE for auxiliary variable $\mathbf{J}$, which is introduced to handle the time convolution resulting from the frequency dependent constitutive