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NUMERICAL LOCALIZATION OF ELECTROMAGNETIC IMPERFECTIONS FROM A PERTURBATION FORMULA IN THREE DIMENSIONS*

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Abstract

This work deals with the numerical localization of small electromagnetic inhomogeneities. The underlying inverse problem considers, in a three-dimensional bounded domain, the time-harmonic Maxwell equations formulated in electric field. Typically, the domain contains a finite number of unknown inhomogeneities of small volume and the inverse problem attempts to localize these inhomogeneities from a finite number of boundary measurements. Our localization approach is based on a recent framework that uses an asymptotic expansion for the perturbations in the tangential boundary trace of the curl of the electric field. We present three numerical localization procedures resulting from the combination of this asymptotic expansion with each of the following inversion algorithms: the Current Projection method, the MUltiple SIgnal Classification (MUSIC) algorithm, and an Inverse Fourier method. We perform a numerical study of the asymptotic expansion and compare the numerical results obtained from the three localization procedures in different settings.

Mathematics subject classification: 35R30, 65N21, 65N30, 78A25.
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1. Introduction

The localization of inhomogeneities contained in a domain is of great importance since it has several practical applications: identification of cancer tumors, detection of anti-personnel mines, localization of cracks,$\cdots$. Usually, when we seek to localize an inhomogeneity contained in a domain, we are concerned with an inverse problem for retrieving the geometry of the inhomogeneity or for imaging the physical parameter that characterizes the heterogeneity of the domain.

Recently, several works have been devoted to the numerical analysis of the localization of inhomogeneities (see, e.g., [3, 5, 6, 10, 24]), in particular in the field of Electrical Impedance Tomography (EIT). The localization model proposed by Cedio-Fengya et al. [10] consists of identifying inhomogeneities of small volume by combining an asymptotic formula with an inversion algorithm. Typically, in [10], the conductivity problem is set in a bounded domain containing a finite number of unknown inhomogeneities of small volume. The inversion algorithm makes use of the asymptotic formula (for perturbations in the voltage potential), and is

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based on a minimization procedure of least-squares type for the calculation of the geometrical parameters of the inhomogeneities (namely the centers and diameters when these are balls for example). Another reconstruction approach of these small conductivity inhomogeneities, also based on a nonlinear minimization procedure, is the one that consists of imaging the electric conductivity in the domain (see, e.g., [3]). Regarding the same conductivity problem, Ammari et al. proposed in [5] a localization process of small inhomogeneities, where the asymptotic formula of [10] is considered for measuring boundary voltage perturbations initiated by electric currents applied on the boundary of the domain. Limited current-to-voltage pairs on the boundary are then used as data of the inversion algorithm which consists, here, of solving a linear system for locating a single inhomogeneity, or of calculating a discrete inverse Fourier transform of a sample of measurements in the case of the localization of multiple inhomogeneities. The inversion algorithm in [5] is then, in contrast to the one of [10], non-iterative and based on one of two linear methods: the Current Projection method (for locating a single inhomogeneity) or the Inverse Fourier method (for locating multiple inhomogeneities).

Volkov formulates in [24] an algorithm based also on the Inverse Fourier method for locating small dielectric inhomogeneities in a two-dimensional bounded domain, from an asymptotic expansion (introduced elsewhere in [5]) for the study of the perturbations in the electric field satisfying the Helmholtz equation. The development of this algorithm is also described in [24] for the identification of three-dimensional dielectric inhomogeneities of small volume, from the far field pattern at a fixed frequency.

In the context of localization in an unbounded domain, Ammari et al. have developed in [1] an algorithm for locating small two-dimensional inclusions buried in a half-space from the scattering amplitude at a fixed frequency. In [1], the continuous problem is set with the help of the two-dimensional Helmholtz equation, an asymptotic expansion of the scattering amplitude is presented, and the inversion algorithm is essentially a method for characterizing the range of a self-adjoint operator. This is a linear method, called MUSIC (MUltiple SIgnal Classification), generally used in signal processing theory, and known for estimating the individual frequencies of multiple-harmonic signals [23].

We refer to [3, 4, 8, 12, 14, 17, 19, 21, 22] for other numerical methods, as well as for tools, aimed at solving the reconstruction problem of conductivity inhomogeneities, elastic inhomogeneities, and dielectric inhomogeneities, in different settings.

More recently, Ammari et al. [6] have introduced a framework for the localization of *three-dimensional electromagnetic* inhomogeneities. This framework considers the time-harmonic Maxwell equations in a three-dimensional bounded domain Ω containing a finite number m of unknown inhomogeneities of small volume, and proposes to localize these inhomogeneities from an asymptotic expansion of the perturbation in the (tangential) boundary magnetic field. In the presence of well-separated inhomogeneities, and also distant from $\partial\Omega$, the boundary of Ω, the asymptotic expansion states that, for any $z \in \partial\Omega$,

$$\begin{aligned}
&(H_\alpha - H_0)(z) \times \nu(z) \,-\, 2\int_{\partial\Omega} \mathrm{curl}_z(\Phi^k(x,z)(H_\alpha - H_0)(x) \times \nu(x)) \times \nu(z)\, d\sigma_x \\
&= 2\alpha^3\omega^2 \sum_{j=1}^{m} \frac{\mu_0}{\mu_j}(\mu_0 - \mu_j) G(z_j, z) \times \nu(z) M^j(\frac{\mu_0}{\mu_j}) H_0(z_j) \\
&\quad + 2\alpha^3 \sum_{j=1}^{m} (\frac{1}{\varepsilon_j} - \frac{1}{\varepsilon_0})((\mathrm{curl}_x\, G)(z_j, z))^T \times \nu(z) M^j(\frac{\varepsilon_0}{\varepsilon_j})(\mathrm{curl}_x\, H_0)(z_j) + \mathcal{O}(\alpha^4)\,. \quad (1.1)
\end{aligned}$$