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AN ADAPTIVE INVERSE ITERATION FEM FOR THE INHOMOGENEOUS DIELECTRIC WAVEGUIDES*

Jianhua Yuan
(*School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China
Email: yuan_bupt@yahoo.com.cn*)

Abstract

We introduce an adaptive finite element method for computing electromagnetic guided waves in a closed, inhomogeneous, pillared three-dimensional waveguide at a given frequency based on the inverse iteration method. The problem is formulated as a generalized eigenvalue problems. By modifying the exact inverse iteration algorithm for the eigenvalue problem, we design a new adaptive inverse iteration finite element algorithm. Adaptive finite element methods based on a posteriori error estimate are known to be successful in resolving singularities of eigenfunctions which deteriorate the finite element convergence. We construct a posteriori error estimator for the electromagnetic guided waves problem. Numerical results are reported to illustrate the quasi-optimal performance of our adaptive inverse iteration finite element method.

Mathematics subject classification: 65N30, 65N25, 78A50.
Key words: Waveguides, Eigenvalue Problem, Inverse Iteration Algorithm, Adaptive Finite Element Method.

1. Introduction

In this paper we consider a closed waveguide defined by a right cylinder with cross section Ω, a bounded, Lipschitz, simply connected polyhedral domain in $\mathbf{R}^2$. The waveguide is filled with an inhomogeneous media whose electromagnetic properties are described by the real-valued functions ε and μ. We assume that the magnetic permeability $\mu = \mu_0$, the magnetic permeability in vacuum, and the dielectric permittivity ε is piecewise constant and has no variation along the waveguide. More precisely, let $\Omega_1 \subset \Omega$ be an open domain, $\Omega_2 = \Omega \backslash \overline{\Omega}_1$. We assume that

$$\varepsilon(x) = \begin{cases} \varepsilon_1 \varepsilon_0 & \text{in} \quad \Omega_1, \\ \varepsilon_2 \varepsilon_0 & \text{in} \quad \Omega_2, \end{cases}$$

where ε_0 is the dielectric permittivity in vacuum.

The waveguide problem is to find solutions to the Maxwell equations which are of the general form

$$\begin{cases} \mathcal{E}(\mathbf{x}, x_3, t) = (\mathbf{E}(\mathbf{x}), E_3(\mathbf{x})) e^{i(\omega t - \beta x_3)}, \\ \mathcal{H}(\mathbf{x}, x_3, t) = (\mathbf{H}(\mathbf{x}), H_3(\mathbf{x})) e^{i(\omega t - \beta x_3)}, \end{cases} \tag{1.1}$$

where $\mathbf{x} = (x_1, x_2) \in \Omega$ and the x_3-axis is along the waveguide, $\omega > 0$ is the angular frequency of the guided wave, β is the constant of propagation, $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field components in the plane of the cross section, and E_3 and H_3 are electric and magnetic components along the waveguide.

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With expression (1.1), the second-order three-dimensional Maxwell equations expressed in terms of the electric field $(\mathbf{E}, E_3)$ reduce to the following two-dimensional equations (cf. e.g. [7, 19, 22]):

$$\begin{cases} \nabla \times (\nabla \times \mathbf{E}) - i\beta \nabla E_3 - (\omega^2 \varepsilon_0 \mu_0)\varepsilon \mathbf{E} + \beta^2 \mathbf{E} = 0, & \text{in} \quad \Omega; \\ -\nabla \cdot (\nabla E_3) - (\omega^2 \varepsilon_0 \mu_0)\varepsilon E_3 - i\beta \nabla \cdot \mathbf{E} = 0, & \text{in} \quad \Omega; \\ \nabla \cdot (\varepsilon \mathbf{E}) - i\beta \varepsilon E_3 = 0, & \text{in} \quad \Omega. \end{cases} \tag{1.2}$$

For simplicity, perfect electric conductor boundary conditions are imposed:

$$\mathbf{E} \times \mathbf{n} = 0, \quad E_3 = 0 \quad \text{ on } \partial\Omega, \tag{1.3}$$

where $\mathbf{n}$ is the unit outer normal to $\partial\Omega$.

Advances in various branches of photonics technologies have established the need for the development of numerical and approximate methods for the analysis of a wide range of waveguide structures that are not amenable to exact analytical studies ([8, 12, 14]). Since no sources are given, (1.2)–(1.3) is an eigenvalue problem. Either ω or β is assumed to be known, and the goal is to find all possible pairs which consist of the other missing constant β or ω and the corresponding field $(\mathbf{E}, E_3)$. Probably the first finite element analysis of the waveguide problems was developed in the 1960s ([14]). Current finite element methods for computing waveguide have been developed by using varied finite element technologies (see,e.g.,[4, 7, 8, 14, 15, 19]). In [7], we investigated the finite element methods over the uniformly refined meshes for this problem in the more physically relevant case, when ω is given, but β is unknown. A similar finite element analysis was recently studied in [19].

For the simplest geometries, the finite element methods reported in [7] and [19] are quasi-optimal. But the structure of the more interesting waveguides encountered in extensive applications is always very complex. With the complex structure, the eigenfunctions of the eigenvalue problem usually display singularities which deteriorate the finite element convergence if uniform mesh refinements are used. We show one such situation in [7] that points to the serious problem. However, adaptive finite element methods based on a posteriori error estimates are known to be successful in resolving this difficulty [5, 20]. Introducing the adaptive finite element method for computing electromagnetic guided waves problemr at a given frequency is our main work in this paper. Compared with the traditional adaptive finite element methods for eigenvalue problems, our method is simpler, since the a posteriori error estimators are easily obtained.

In this paper, we will discuss the adaptive finite element algorithm for computing electromagnetic guided waves in a closed, inhomogeneous, pillared three-dimensional waveguide at a given frequency based on the finite element formulation given by [7] and [19] (see [22] for more details).

Let $\mathbb{X} = H_0(\mathbf{curl}; \Omega) \times H_0^1(\Omega)$ equipped with the norm

$$\|(\mathbf{V}, q)\|_{\mathbb{X}} = \| \mathbf{V} \|_{\mathbf{curl},\Omega} + \| q \|_{H^1(\Omega)} \quad \forall \ (\mathbf{V}, q) \in \mathbb{X}.$$

Here,

$$\| \mathbf{V} \|_{\mathbf{curl},\Omega} = (\| \nabla \times \mathbf{V} \|^2_{L^2(\Omega)} + \| \mathbf{V} \|^2_{L^2(\Omega)})^{1/2}$$

is the norm of the space $H(\mathbf{curl}; \Omega)$ which is defined as the collection of all functions $\mathbf{V}$ in $L^2(\Omega)$ such that $\| \mathbf{V} \|_{\mathbf{curl},\Omega} < \infty$. $H_0(\mathbf{curl}; \Omega)$ consists of functions $\mathbf{V}$ in $H(\mathbf{curl}; \Omega)$ whose tangential component $\mathbf{V} \times \mathbf{n}$ vanishes on the boundary $\partial\Omega$.

Let $E_3^{\text{new}} = i\beta E_3$. To save the notation, E_3 will represent E_3^{new} for the remainder of this paper. Similar to in [7], [19] and [22], our goal is to solve the following variational problem for