Journal of Computational Mathematics, Vol.24, No.5, 2006, 591–608.

A NEW CONSTRAINTS IDENTIFICATION TECHNIQUE-BASED QP-FREE ALGORITHM FOR THE SOLUTION OF INEQUALITY CONSTRAINED MINIMIZATION PROBLEMS [*1)]

Chang-yin Zhou
(*Department of Mathematics, Shanghai Jiao Tong University, Shanghai 200240, China; College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao 266510, China*)

Guo-ping He Yong-li Wang
(*College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao 266510, China*)

Abstract

In this paper, we propose a feasible QP-free method for solving nonlinear inequality constrained optimization problems. A new working set is proposed to estimate the active set. Specially, to determine the working set, the new method makes use of the multiplier information from the previous iteration, eliminating the need to compute a multiplier function. At each iteration, two or three reduced symmetric systems of linear equations with a common coefficient matrix involving only constraints in the working set are solved, and when the iterate is sufficiently close to a KKT point, only two of them are involved. Moreover, the new algorithm is proved to be globally convergent to a KKT point under mild conditions. Without assuming the strict complementarity, the convergence rate is superlinear under a condition weaker than the strong second-order sufficiency condition. Numerical experiments illustrate the efficiency of the algorithm.

Mathematics subject classification: 90C30, 65K10.
Key words: QP-free method, Optimization, Global convergence, Superlinear convergence, Constraints identification technique.

1. Introduction

Consider the following inequality constrained optimization problem.

$$min \;\; f(x) \qquad subject \;\; to \qquad c(x) \leq 0 \tag{1.1}$$

where $f(\cdot) : R^n \to R$ and $c(\cdot) : R^n \to R^m$ are twice continuously differentiable functions. Define $\mathcal{F}:= \{x \in R^n \mid c(x) \leq 0\}$ and let $I = \{1, \cdots, m\}$. For any $x \in\mathcal{F}$, the active set is denoted by $I_0(x) = \{i \in I | c_i(x) = 0\}$.

It is well known that sequential quadratic programming(SQP) methods is one of the most efficient methods for solving nonlinear constrained optimization problems. Under certain conditions SQP methods also possess good global and superlinear convergence properties. Therefore, SQP methods have received much attention in recent decades. However, there are still some defaults existed in SQP methods. For example, the QP subproblem may be inconsistent, that is, the feasible set of the QP subproblem may be empty. Moreover, it is usually difficult to use some good spare and symmetric properties of the original problem, which may restrict the application of the SQP algorithm, especially for large-scale problems. We refer to the review paper [1] for an excellent survey about SQP methods.

[*] Received May 30, 2004; final revised December 9, 2005.
[1)] This work is supported by the National Natural Science Foundation of China (10571109).

Sequential systems of linear equations(SSLE in short) method, also called QP-free method, is proposed mainly to overcome the shortcomings of SQP method mentioned above. Panier, Tits and Herskovits [2] first proposed a feasible QP-free algorithm for solving problem (1.1). At each iteration, they first solve two systems of linear equations with the following form

$$\begin{pmatrix} H_k & A(x^k) \\ MA(x^k)^T & G(x^k) \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ \triangle_k \end{pmatrix} \tag{1.2}$$

where $A(x^k) = (\nabla c_1(x^k), \cdots, \nabla c_m(x^k))^T$, $G(x^k) = diag((c_i(x^k))$ and $M = diag(\mu_i)$. $diag((c_i(x^k))$ and $diag(\mu_i)$ denote the $m \times m$ diagonal matrix whose i-th diagonal element are $c_i(x^k)$ and $\mu_i (i = 1, \cdots, m)$, respectively. To avoid the Maratos effect, locally a second order correction is computed by solving a least squares subproblem. It is shown that any accumulation point of the iterates generated by the algorithm is a stationary point of problem (1.1). If further the stationary points are assumed to be finite, the point is proved to be a KKT point. It is also shown that the algorithm has a two-step superlinear convergence rate. Later, Z. Gao, G. He and F. Wu[4] improved the algorithm in the sense that any accumulation point is a KKT point without assuming the isolatedness of the accumulation point. To achieve this, they solve an extra linear system obtained from (1.2) by slightly perturbing the right-hand side of (1.2). Recently, their algorithm is further improved in [5] by themselves, which proved the one-step superlinear convergence under the strict complementarity and second order sufficient condition. The same idea is also used to develop a primal-dual logarithmic barrier interior-point method by Urban, Tits and Lawrence [3] and an infeasible SSLE algorithm for solving general constrained optimization problems in [6]. However, the coefficient matrix in linear system (1.2) may become ill-conditioned if the multiplier μ_i corresponding to a nearly active constraint $c_i(x)$ becomes very small. This may easily occur if the strict complementarity doesn't hold at the solution of problem (1.1). To avoid the ill-conditionedness, H. Qi and L. Qi [10] proposed a new QP-free algorithm for solving problem (1.1), that is based on a nonsmooth equation reformulation of the KKT system of problem (1.1) by using the Fisher-Burmeister function. It is shown that under the Linear Independence Constraint Qualification, the coefficient matrix in [10] is uniformly nonsingular and well-conditioned even if the strict complementarity does not hold at the accumulation point. But they still need strict complementarity to prove the superlinear convergence of their algorithm.

On the other hand, Z. Gao, G. He and F. Wu [7] proposed an infeasible SSLE method for general constrained optimization problems. At each iteration, they solve three symmetric systems of linear equations of the following form

$$\begin{pmatrix} H_k & \nabla c_{I_k}(x^k) \\ \nabla c_{I_k}(x^k)^T & 0 \end{pmatrix} \begin{pmatrix} d \\ \lambda \end{pmatrix} = \begin{pmatrix} -\nabla f(x^k) \\ \triangle_k \end{pmatrix} \tag{1.3}$$

where $I_k \subseteq I$ is a working set and an estimate of the active set $I_0(x^k)$. The same idea is also used in [8] by Y.F. Yang, D.H. Li and L. Qi to develop a feasible QP-free algorithm for solving problem (1.1), based on an active constraints identification technique proposed by Facchinei, Fischer and Kanzow [9]. Different from algorithms in [2-6,10], algorithms in [7,8] are not affected by the ill-conditionedness of coefficient matrix triggered by dual degeneracy. However, in order to prove superlinear convergence, they still need strict complementarity to hold. Other similar QP-free methods can also be seen in [11-13].

Ever since the first QP-free algorithm was proposed, it has always been an important research area to establish superlinear convergence without strict complementarity. Facchinei and Lucidi[14], Bonnans[15] have proposed several local QP-free algorithms whose rapid convergence does not need the strict complementarity to hold. Facchinei , Lucidi and Palagi[16] proposed a globally convergent truncated Newton method for solving box constrained optimization problem, whose superlinear convergence also does not rely on the strict complementarity. However, globally convergent QP-free algorithm for generally constrained optimization problems without