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A LQP BASED INTERIOR PREDICTION-CORRECTION METHOD FOR NONLINEAR COMPLEMENTARITY PROBLEMS *1)

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Abstract

To solve nonlinear complementarity problems (NCP), at each iteration, the classical proximal point algorithm solves a well-conditioned sub-NCP while the Logarithmic-Quadratic Proximal (LQP) method solves a system of nonlinear equations (*LQP system*). This paper presents a practical LQP method-based prediction-correction method for NCP. The predictor is obtained via solving the *LQP system* approximately under significantly relaxed restriction, and the new iterate (the corrector) is computed directly by an explicit formula derived from the original LQP method. The implementations are very easy to be carried out. Global convergence of the method is proved under the same mild assumptions as the original LQP method. Finally, numerical results for traffic equilibrium problems are provided to verify that the method is effective for some practical problems.

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1. Introduction

The nonlinear complementarity problem (NCP) is to determine a vector $x \in R^n$ such that

$$x \geq 0, \quad F(x) \geq 0 \quad \text{and} \quad x^T F(x) = 0, \tag{1.1}$$

where F is a nonlinear mapping from R^n into itself. NCP has received a lot of attention due to its various applications in operations research, economic equilibrium, engineering design, and others, e.g., [7, 8].

A classical method for solving NCP is the Proximal Point Algorithm (PPA) proposed first by Martinet [12] and then developed by many researchers, e.g., [6, 9, 15, 16]. For given $x^k \in R^n_+$ and $\beta_k > 0$, the new iterate x^{k+1} generated by PPA is the unique solution of the following auxiliary NCP: Find $x \in R^n$ such that

$$x \geq 0, \quad \beta_k F(x) + (x - x^k) \geq 0 \quad \text{and} \quad x^T(\beta_k F(x) + (x - x^k)) = 0. \tag{1.2}$$

Recently, a number of articles have concentrated on the generalization of PPA by replacing the linear term $x - x^k$ with some nonlinear functions $r(x, x^k)$. As a result, some "interior point" proximal methods for variational inequality problems have been developed by introducing entropic proximal terms arising from appropriately formulated Bregman functions [1, 4, 5, 6]

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and entropic φ-divergence [16]. For given $x^k \in R^n_{++} := \text{int}R^n_+$ and $\beta_k > 0$, the Logarithmic-Quadratic Proximal (LQP) method presented by Auslender, *et al.* in [2] takes the unique solution of the following auxiliary NCP as the new iterate:

$$x \geq 0, \quad \beta_k F(x) + \nabla_x D(x, x^k) \geq 0 \quad \text{and} \quad x^T(\beta_k F(x) + \nabla_x D(x, x^k)) = 0, \tag{1.3}$$

where

$$\nabla_x D(x, x^k) = (x - x^k) + \mu(x^k - X_k^2 x^{-1}), \tag{1.4}$$

μ is a parameter in $(0, 1)$, $X_k = \text{diag}(x_1^k, x_2^k, \ldots, x_n^k)$ and x^{-1} is an n-vector whose j-th element is $1/x_j$. Note that the integral function of $\nabla_x D(x, x^k)$ satisfying $D(x^k, x^k) = 0$ is

$$D(x, x^k) = \begin{cases} \frac{1}{2}\|x - x^k\|^2 + \mu \sum_{j=1}^n \Big((x_j^k)^2 \log \frac{x_j^k}{x_j} + x_j x_j^k - (x_j^k)^2 \Big), & \text{if } x \in \mathcal{R}^n_{++}, \\ +\infty & \text{otherwise.} \end{cases} \tag{1.5}$$

Since $D(x, x^k)$ includes logarithmic and quadratic terms, the method is called *Logarithmic-Quadratic Proximal method*. The first term of $\nabla_x D(x, x^k)$ is to avoid that the new iterate is too far away from x^k; and the second term is to guarantee that the new iterate lies in R^n_{++}. Therefore, at the k-th iteration, solving NCP by the LQP method is equivalent to finding the positive solution of the following system of nonlinear equations

$$\beta_k F(x) + x - (1 - \mu)x^k - \mu X_k^2 x^{-1} = 0. \tag{1.6}$$

Throughout this paper, we call (1.6) the *LQP system of nonlinear equations* (abbreviated as *LQP system*). Generally speaking, solving the *LQP system* is much easier than solving the auxiliary NCP (1.2). Thus the LQP method is attractive for solving NCP. In general, however, it is not trivial to obtain the exact positive solution of the *LQP system*. An inexact LQP method solving (1.6) approximately was also presented in [2].

In this paper, inspired by the LQP method, we present a prediction-correction method [11] for NCP. Both the predictor and corrector are computed via an explicit formula derived from (1.6) (For details, see (2.1) and (2.3)). Similar to the LQP method, all the iterative points generated by the method lie in R^n_{++} whenever the initial point does. Thus the method inherits theoretical properties of the original LQP method. Based on these observations, we call the method a LQP based interior prediction-correction method.

The rest of this paper is organized as follows. In Section 2, the new method is presented and some remarks are also provided. In Section 3, we prove the contractive properties of the proposed method. These properties play important roles in the convergence analysis. Convergence of the new method is discussed in Section 4. In Section 5, some implementation details of the proposed method are addressed. In addition, numerical results for problems in traffic equilibrium are also reported. Finally, some conclusions are drawn in Section 6.

Throughout this paper we make the following standard assumptions:

A1. $F(x)$ is continuous and monotone mappings with respect to R^n_+, i.e.,

$$(x - \tilde{x})^T(F(x) - F(\tilde{x})) \geq 0, \quad \forall x, \tilde{x} \in R^n_+. \tag{1.7}$$

A2. The solution set of the NCP, denoted by $\mathcal{X}^*$, is nonempty.

2. The Proposed Method

At the k-th iteration, the LQP method solves the *LQP system* (1.6) exactly or approximately. We now present a LQP based interior prediction-correction method for NCP.

Let $\mu, \eta \in (0, 1)$. For given $x^k > 0$ and $\beta_k > 0$, the new iterate x^{k+1} is generated by the following steps:

Prediction step: Find an approximate solution $\tilde{x}^k$ of (1.6), called predictor, such that

$$0 \approx \beta_k F(\tilde{x}^k) + \tilde{x}^k - (1 - \mu)x^k - \mu X_k^2 (\tilde{x}^k)^{-1} =: \xi^k, \tag{2.1}$$