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AN OPTIMAL METHOD FOR ADJUSTING THE CENTERING PARAMETER IN THE WIDE-NEIGHBORHOOD PRIMAL-DUAL INTERIOR-POINT ALGORITHM FOR LINEAR PROGRAMMING *1)

Wen-bao Ai
(*Department of Mathematics, Beijing University of Posts and Telecommunications, Beijing 100876, China*)

Abstract

In this paper we present a dynamic optimal method for adjusting the centering parameter in the wide-neighborhood primal-dual interior-point algorithms for linear programming, while the centering parameter is generally a constant in the classical wide-neighborhood primal-dual interior-point algorithms. The computational results show that the new method is more efficient.

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1. Introduction

The primal problem and the dual problem we are concerned with are

(LP) $\min\ c^T x,\ \text{s.t.}\ \ Ax = b,\ x \geq 0,$
(LD) $\max\ b^T y,\ \text{s.t.}\ \ A^T y + s = c,\ s \geq 0,$

where $A \in R^{m\times n},\ c, x, s \in R^n$ and $y, b \in R^m$. Let

$$F_{++} = \{(x,s) \in R^{2n} : Ax = b,\ A^T y + s = c \ \text{ for some } y \in R^m,\ (x,s) > 0\}.$$

We say that the point (x,s) is an interior feasible pair if $(x,s) \in F_{++}$. We also use the notation $w := (x,s)$.

Let us look back the iteration process of the primal-dual interior point algorithms: Assume that (x,s) is a current iterate interior feasible pair. Let X and S denote the diagonal matrix obtained from the vectors x and s respectively, i.e. $X = diag(x)$ and $S = diag(s)$. Then, the search direction of the typical primal-dual interior point methods is obtained by solving the following linear equations system:

$$\begin{array}{lcll} A\Delta x & = & 0, & \\ A^T\Delta y + \Delta s & = & 0, & \\ S\Delta x + X\Delta s & = & \gamma\mu e - Xs, & \end{array} \tag{1.1}$$

where $\mu = x^T s/n$ and $\gamma \in [0,1]$ is so-called centering parameter. The solution of (1.1) is denoted by $\Delta w(\gamma) = (\Delta x(\gamma), \Delta s(\gamma))$. Then the next iteration pair $w(\gamma,\theta) = (x(\gamma,\theta), s(\gamma,\theta))$ is obtained by setting

$$\begin{aligned} x(\gamma,\theta) &= x + \theta\Delta x(\gamma), \\ s(\gamma,\theta) &= s + \theta\Delta s(\gamma), \end{aligned}$$

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or in a more compact notation

$$w(\gamma,\theta) = w + \theta\Delta w(\gamma),$$

where $\theta \in (0,+\infty)$ is chosen so that $w(\gamma,\theta) \geq 0$ or $w(\gamma,\theta)$ belongs to some neighborhood of the centering path. Two choices of γ have special interest:

(i) $\gamma = 0$: the affine scaling direction $\Delta w^a = (\Delta x^a, \Delta s^a)$;

(ii) $\gamma = 1$: the centering scaling direction $\Delta w^c = (\Delta x^c, \Delta s^c)$.

By superposition, the search direction (1.1) satisfies

$$\Delta w(\gamma) = \gamma\Delta w^c + (1-\gamma)\Delta w^a. \tag{1.2}$$

and the resulting point will be

$$w(\gamma,\theta) = w + \theta\gamma\Delta w^c + \theta(1-\gamma)\Delta w^a. \tag{1.3}$$

Obviosly, the value of γ plays an important role in the primal-dual interior point algorithms. Generally, the typical choices of the centering parameter γ are as follows:

(1) In narrow neighborhood path-following methods([8, 4, 1]), γ is the smallest value such that

$$\|X(\gamma,1)s(\gamma,1) - \mu(\gamma,1)e\| \leq 0.25\mu(\gamma,1),$$

where $\mu(\gamma,1) = x(\gamma,1)^T s(\gamma,1)/n$.

(2) In narrow neighborhood predictor-corrector methods([7]), $\gamma = 0$ and $\gamma = 1$ respectively.

(3) In wide neighborhood methods ([3, 2, 6, 9]), γ is often a small positive constant between 0.001 and 0.005.

We have to remark that, although the methods in (1) and (2) have the best iteration bound $O(\sqrt{n})$ theoretically, they are less practical; and although theoretical iteration bound of the methods in wide neighborhoods (3) is not best (only $O(n)$), they are more practical because they allow long steps, which is a prerequisite for practical efficiency. Notice that γ is a constant for all iterations in wide neighborhood methods, which is not the best choice obviously. It is surely true that practical efficiency will be better if γ can be adaptively adjusted according to the current iteration interior pair. This is really purpose of the paper. In section 2, we introduce a dynamic optimal adjusting method of γ according to the current iteration interior pair. In section 3, we give our algorithms and its convergence proof. Finally, in section 4, we show our computational results.

In this paper, we often use the following conclusion that is well-known and easily proved:

Conclusion A. *For any $\Delta x \in R^n$ and $\Delta s \in R^n$ that satisfy $A\Delta x = 0$ and $A^T\Delta y + \Delta s = 0$, there is*

$$\Delta x^T \Delta s = 0.$$

We also need the terminology $N_\infty^-(\eta)$ from Mizuno, Todd and Ye ([7]), which is

$$N_\infty^-(\eta) := \{(x,s) \mid (x,s) \in F_{++}, Xs \geq (1-\eta)\mu e\},$$

where $\eta \in (0,1)$ and $\mu = x^T s/n$.

2. An Optimal Method for Adjusting γ

Suppose that (x,s) is a current iterate interior feasible pair. We choose a centering parameter γ such that the duality gap of the new iteration pair $w(\gamma)$ would minimized, i.e. γ is a solution of the following optimization problem:

$$\min\ x(\gamma,\theta)^T s(\gamma,\theta) \qquad \text{s.t.}\ \ x(\gamma,\theta) \geq 0\ \ \text{and}\ \ s(\gamma,\theta) \geq 0. \tag{2.1}$$