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THE ARTIFICIAL BOUNDARY CONDITIONS FOR NUMERICAL SIMULATIONS OF THE COMPLEX AMPLITUDE IN A COUPLED BAY-RIVER SYSTEM *1)

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Abstract

We consider the numerical approximations of the complex amplitude in a coupled bay-river system in this work. One half-circumference is introduced as the artificial boundary in the open sea, and one segment is introduced as the artificial boundary in the river if the river is semi-infinite. On the artificial boundary a sequence of high-order artificial boundary conditions are proposed. Then the original problem is solved in a finite computational domain, which is equivalent to a variational problem. The numerical approximations for the original problem are obtained by solving the variational problem with the finite element method. The numerical examples show that the artificial boundary conditions given in this work are very effective.

Mathematics subject classification: 65N05, 76B20.
Key words: Coupled bay-river system, Complex amplitude, Artificial boundary conditions, Finite element method.

1. Introduction

This paper is to solve the two-dimensional complex amplitude in a coupled bay-river system. Consider a bay located at a river mouth and connected to the open sea through a narrow entrance(see Figure 1). Such a coupled bay-river system, may be agitated into resonant states by external oscillations with particular periods. The water depth everywhere in the domain is assumed to be constant, and the solid boundary is considered to be impermeable. The model river possesses an invariant cross section, and the length of river could be either semi-infinite or finite. In these two cases, the non-reflective and perfect reflective conditions will be used in the end of the river respectively. Additionally, the width of the river and the bay entrance are assumed to be small compared with both the dimension of the bay and the wavelength.

The river mouth , bay entrance and coastline are defined by

$$\begin{aligned}
\Gamma_{rm} &= \{(-a,y)| -\frac{t}{2} < y < \frac{t}{2}\},\\
\Gamma_{bs} &= \{(0,y)| -\frac{\epsilon}{2} < y < \frac{\epsilon}{2}\},\\
\Gamma_{cl} &= \{(0,y)||y| > \frac{\epsilon}{2}\}.
\end{aligned}$$

Let Ω_r , Ω_b and Ω_s denote the domain occupied by the river , the bay and the open sea respectively. Then we have

$$\begin{aligned}
\Omega_r &= \{(x,y)| -s-a < x < -a, -\frac{t}{2} < y < \frac{t}{2}\},\\
\Omega_s &= \{(x,y)|x > 0\};
\end{aligned}$$

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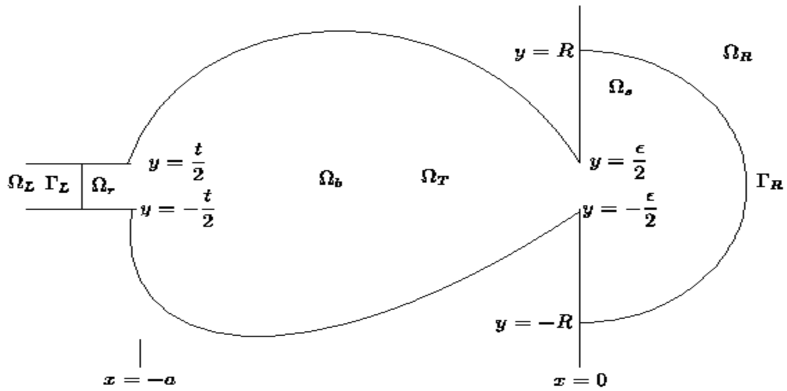

Figure 1: The physical and the computational domain

where s is the length of the river. $s = 0$ represents no river and $s = \infty$ represents a semi-infinite river.

In the following, we only discuss the case $s = \infty$.

The Ω_b can have any shape, Figure 1 shows one applicable choice.

Denote $\Omega_{br} = \Omega_r \cup \Omega_b \cup \Gamma_{rm}$, and $\Gamma_{br} = \partial\Omega_{br} \setminus \Gamma_{bs}$.

We have the following equations and boundary conditions for the complex amplitude of water surface oscillation(cf. [17]):

$$\triangle\eta + (1+i\xi)^2 k^2 \eta = 0 \qquad \text{in } \Omega_{br}, \tag{1.1}$$

$$\triangle\eta + k^2\eta = 0 \qquad \text{in } \Omega_s, \tag{1.2}$$

$$\frac{\partial\eta}{\partial n} = 0 \qquad \text{on } \Gamma_{br}, \tag{1.3}$$

$$\frac{\partial\eta}{\partial n} = 0 \qquad \text{on } \Gamma_{cl}, \tag{1.4}$$

$$\eta, \frac{\partial\eta}{\partial x} \text{ is continuous} \qquad \text{on } \Gamma_{\text{bs}}, \tag{1.5}$$

$$\lim_{r\to\infty} \sqrt{r}[\frac{\partial(\eta-\eta_0)}{\partial r} - ik(\eta-\eta_0)] = 0, \tag{1.6}$$

$$\begin{aligned}&\text{In the river } \eta \text{ is bounded and}\\ &\text{represents waves that}\\ &\text{propagate in the negative x} - \text{direction;}\end{aligned} \tag{1.7}$$

where η is the complex amplitude of water surface oscillation, of which the modulus and the argument represent the conventional amplitude and the relative phase of the local water surface oscillations, respectively; k denotes wave number; ξ is the resistance coefficient to model the effects of dissipation, in the open sea, dissipativity is generally insignificant, so (1.2) takes (1.1) when $\xi = 0$; $\frac{\partial}{\partial n}$ represents outward normal derivative of $\partial\Omega_{br}$ or $\partial\Omega_s$; $r = \sqrt{x^2+y^2}$ when $x > 0$, condition (1.6) is known as the Sommerfeld radiation condition; η_0 represents the water surface oscillation induced by the incident wave along with the coastline reflection in case of a vanishing bay entrance and, consequently, a continuous coastline. If the incident wave is of a wave height H_0 and is in the direction forming an angle α with the x-axis, then η_0 can be