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CONVERGENCE OF NEWTON'S METHOD FOR A MINIMIZATION PROBLEM IN IMPULSE NOISE REMOVAL *1)

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Dedicated to Professor Zhong-ci Shi on the occasion of his 70th birthday

Abstract

Recently, two-phase schemes for removing salt-and-pepper and random-valued impulse noise are proposed in [6, 7]. The first phase uses decision-based median filters to locate those pixels which are likely to be corrupted by noise (noise candidates). In the second phase, these noise candidates are restored using a detail-preserving regularization method which allows edges and noise-free pixels to be preserved. As shown in [18], this phase is equivalent to solving a one-dimensional nonlinear equation for each noise candidate. One can solve these equations by using Newton's method. However, because of the edge-preserving term, the domain of convergence of Newton's method will be very narrow. In this paper, we determine the initial guesses for these equations such that Newton's method will always converge.

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1. Introduction

Impulse noise is caused by malfunctioning pixels in camera sensors, faulty memory locations in hardware, or transmission in a noisy channel. Some of the pixels in the images could be corrupted by the impulse noise while the remaining pixels remain unchanged. There are two types of impulse noise: fixed-valued noise and random-valued noise. For images corrupted by fixed-valued noise, the noisy pixels can take only some of the values in the dynamic range, e.g. the maximum and the minimum values in the so-called salt-and-pepper noise model. In contrast, the noisy pixels in images corrupted by random-valued noise can take any random values in the dynamic range.

There are many works proposed to clean the noise, see for instance the schemes proposed in [2, 17, 1, 12, 13, 19, 18, 6, 7]. In particular, decision-based median filters are popular in removing impulse noise because of their good denoising power and computational efficiency, see [16, 15, 22, 9, 20, 14]. However, the blurring of details and edges are clearly visible when the noise level is high. In comparison, the detail-preserving variational method proposed in [18] used non-smooth data-fitting term along with edge-preserving regularization to restore the images. The variational method can keep the edges. But when removing noise patches—several noise pixels connecting each other, the distortion of some uncorrupted image pixels at the edges

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cannot be avoided. To overcome the drawbacks, the two-phase schemes recently proposed in [6, 7] combine decision-based median filters and the detail-preserving variational method to clean the noise.

The first phase in the methods proposed in [6, 7] is based on the adaptive median filter [15] or the adaptive center-weighted median filter [9] to first locate those pixels which are likely to be corrupted by noise (noise candidates). Because of computational efficiency of median filters, this phase can be processed in a short time. The second phase is to restore those noise candidates by variational method given in [18]. It is to minimize the objective functional consisting of a data-fitting term and an edge-preserving regularization term. It is equivalent to solving a system of nonlinear equations for those noise candidates. As shown in [18], the root finding can be done by relaxation, and it results in solving a one-dimensional nonlinear equation for each noise candidate. The presence of the edge-preserving regularization term introduces difficulties in solving the equations because the nonlinear functions can have very large derivatives in some regions. In particular, the convergence domain can be very small if Newton's method is used. In this report, we give an algorithm to locate the initial guess such that Newton's method always converges.

The outline of this report is as follows. In §2, we review both two-phase denoising schemes proposed in [6] and [7] for cleaning impulse noises. The initial guess of Newton's method for solving nonlinear equations is discussed in §3. Numerical results and conclusions are presented in §4 and §5 respectively.

2. Review of 2-Phase Denoising Schemes

Let $\{x_{ij}\}_{i,j=1}^{M,N}$ be the gray level of a true image $\mathbf{x}$ at pixel location (i,j), and $[s_{\min}, s_{\max}]$ be the dynamic range of $\mathbf{x}$. Denote $\mathbf{y}$ the noisy image. The observed gray level at pixel location (i,j) is given by

$$y_{ij} = \begin{cases} r_{ij}, & \text{with probability } p, \\ x_{ij}, & \text{with probability } 1-p, \end{cases}$$

where p defines the noise level. In salt-and-pepper noise model, r_{ij} take either $s_{\min}$ or $s_{\max}$, i.e. $r_{ij} \in \{s_{\min}, s_{\max}\}$, see [15]. In random-valued noise model, $r_{ij} \in [s_{\min}, s_{\max}]$ are random numbers, see [9].

2.1. Cleaning Salt-and-pepper Noise

A two-phase scheme is proposed in [6] to remove salt-and-pepper noise. The first phase is to use the adaptive median filter (AMF) [15] to identify the noise candidates. Then the second phase is to restore those noise candidates by minimizing the objective functional proposed in [18] which consists of an ℓ_1 data-fitting term and an edge-preserving regularization term. The algorithm is as follows:

Algorithm I.

1. *(Noise detection):* Apply AMF to the noisy image $\mathbf{y}$ to get the noise candidate set $\mathcal{N}$.

2. *(Refinement):* If the range of the noise is known, we can refine $\mathcal{N}$ to $\mathcal{N}_T$. For example,

$$\mathcal{N}_T = \mathcal{N} \cap \{(i,j): \ s_{\min} \le y_{ij} \le s_{\min} + T \quad \text{or} \quad s_{\max} - T \le y_{ij} \le s_{\max}\},$$

where $T \ge 0$ is a threshold. Or we can choose T such that

$$\frac{|\mathcal{N}_T|}{M \times N} \approx p.$$

In the case of salt-and-pepper noise we can take T close to zero.