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EXPERIMENTAL STUDY OF THE ASYNCHRONOUS MULTISPLITTING RELAXATION METHODS FOR THE LINEAR COMPLEMENTARITY PROBLEMS*[1)]

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Abstract

We study the numerical behaviours of the relaxed asynchronous multisplitting methods for the linear complementarity problems by solving some typical problems from practical applications on a real multiprocessor system. Numerical results show that the parallel multisplitting relaxation methods always perform much better than the corresponding sequential alternatives, and that the asynchronous multisplitting relaxation methods often outperform their corresponding synchronous counterparts. Moreover, the two-sweep relaxed multisplitting methods have better convergence properties than their corresponding one-sweep relaxed ones in the sense that they have larger convergence domains and faster convergence speeds. Hence, the asynchronous multisplitting unsymmetric relaxation iterations should be the methods of choice for solving the large sparse linear complementarity problems in the parallel computing environments.

Key words: Linear complementarity problem, Matrix multisplitting, Asynchronous iterative methods, Numerical experiment.

1. Introduction

Given a matrix $M = (m_{kj}) \in \mathbb{R}^{n\times n}$ and a vector $q = (q_k) \in \mathbb{R}^n$, the linear complementarity problem LCP(M, q) is to find a vector $z \in R^n$ such that

$$Mz + q \geq 0, \quad z \geq 0 \quad \text{and} \quad z^T(Mz + q) = 0.$$

Recently, many practical and efficient parallel iterative methods in the sense of matrix multisplitting were proposed for solving the LCP(M, q) on the high-speed multiprocessor systems, and the convergence properties of these methods were studied in depth for some standard matrix classes. For details we refer to [4] and references therein. In original, these methods were developed from the matrix multisplitting iterative methods for the system of linear equations (see Bai [1] and Bai, Sun and Wang [7]), as well as from the sequential iterative methods for solving the linear complementarity problems (see Cottle, Pang and Stone [9]).

Based on several splittings of the system matrix $M \in \mathbb{R}^{n\times n}$, the LCP$(M, q)$ can be decomposed into independent linear complementarity problems of smaller sizes. Through solving these sub-problems in parallel on the multiprocessor system without any communication barrier, Bai and Huang [6] presented an asynchronous multisplitting iterative method. This asynchronous multisplitting iterative method was established in accordance with the principle of using sufficiently and communicating flexibly the known information. Hence, it has the potential to achieve high parallel computing efficiency in actual computations. For the convenience

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of real applications, Bai and Huang [6] further presented an explicit alternative, called the asynchronous multisplitting unsymmetric AOR method, of the above mentioned asynchronous multisplitting iterative method, by making use of the overrelaxation and acceleration techniques. This asynchronous multisplitting unsymmetric AOR method includes two relaxation sweeps within each of its iterations, and each sweep possibly includes its own pair of relaxation parameters. Therefore, it can cover a series of relaxed asynchronous multisplitting methods for solving the LCP(M,q). Moreover, a numerical example was given in [6] to show that these relaxed asynchronous multisplitting methods are quite efficient for solving the large sparse linear complementarity problems on the high-speed multiprocessor systems.

In this paper, we further study the numerical behaviours of the relaxed asynchronous multisplitting methods by solving some typical problems from practical applications. For various choices of the relaxation parameters and in both of sequential and parallel settings, a variety of experiments were implemented for the asynchronous multisplitting unsymmetric AOR method and its synchronous and sequential alternatives, as well as some of their typical cases from special choices of the relaxation parameters. Numerical results show that the parallel multisplitting relaxation methods always perform much better than their corresponding sequential alternatives, and that the asynchronous multisplitting relaxation methods often outperform their synchronous counterparts. Moreover, the two-sweep relaxed multisplitting methods have better convergence properties than the corresponding one-sweep relaxed ones in the sense that they have larger convergence domains and faster convergence speeds. Hence, the asynchronous multisplitting unsymmetric relaxation iterations should be the methods of choice for solving the large sparse linear complementarity problems in the parallel computing environments.

2. The Relaxed Asynchronous Multisplitting Methods

We assume that the considered multiprocessor system consists of α processors, and the host processor may be chosen to be any one of them. For a matrix $M \in \mathbb{R}^{n\times n}$, let $M = B_i + C_i$ $(i = 1,2,\dots,\alpha)$ be α Q-splittings (see [2, 3, 9, 11]) and $E_i \in \mathbb{R}^{n\times n}$ $(i = 1,2,\dots,\alpha)$ be α nonnegative diagonal matrices satisfying $\sum_{i=1}^{\alpha} E_i = I$ (the $n\times n$ identity matrix). Then the collection of triples $(B_i, C_i, E_i)(i = 1,2,\dots,\alpha)$ is called a multisplitting of the matrix M, and the matrices $E_i(i = 1,2,\dots,\alpha)$ are called weighting matrices. We introduce the following necessary notations for describing an asynchronous multisplitting iteration: $N_0 = \{0,1,2,\dots\}$; for $\forall p \in N_0$, $J(p)$ is a nonempty subset of the number set $\Lambda = \{1,2,\dots,\alpha\}$; and for $\forall i \in \Lambda$ and $\forall p \in N_0$, $s_i(p)$ is an infinite sequence of nonnegative integers, such that: (1) for $\forall i \in \Lambda$, the set $\{p \in N_0 | i \in J(p)\}$ is infinite; (2) for $\forall i \in \Lambda$ and $\forall p \in N_0$, it holds that $s_i(p) \le p$; and (3) for $\forall i \in \Lambda$, it holds that $\lim_{p\to\infty} s_i(p) = \infty$. If we denote $s(p) = \min_{1\le i\le\alpha} s_i(p)$, then it holds that $s(p) \le p$ and $\lim_{p\to\infty} s(p) = \infty$. Assumption (1) demands that all processors of the multiprocessor system must proceed their local iterations without dead breakdown, Assumption (2) demands that the currently unavailable information should not be used in the current computations, and Assumption (3) demands that every processor of the multiprocessor system must adopt new information to update its local variables continually.

Let $(B_{p,i}, C_{p,i}, E_i)(i = 1,2,\dots,\alpha)$, $p \in N_0$, be a sequence of multisplittings of the matrix M. Then the following asynchronous multisplitting relaxation method for solving the LCP(M,q) was presented in Bai and Huang [6]:

Method 2.1. (Asynchronous Multisplitting Relaxation Method).
Given an initial vector $z^0 \in \mathbb{R}^n$. Suppose that we have obtained the approximate solutions $z^1, z^2, \dots, z^p$ of the LCP(M,q). Then the next approximate solution z^{p+1} of the LCP(M,q) is