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A FIFTH-ORDER ACCURATE WEIGHTED ENN DIFFERENCE SCHEME AND ITS APPLICATIONS*[1)]

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Abstract

In this paper, we have constructed a high accurate difference scheme based on the ENN scheme [1]. The new scheme has 5th-order accuracy in smooth regions and can keep the essencially non-oscillatory property.

Key words: ENN scheme, WENO scheme, Shock-boundary-layer interaction.

1. Introduction

In the paper [1] , where Zhang Hanxin et al. presented the nonoscillatory 3rd-order ENN difference scheme. The idea of ENN scheme is to compare the 1st-order difference and 2nd-order difference to attain 3rd-order accurate scheme and to avoid spurious oscillations near shocks. However the ENN scheme has certain drawbacks. One problem is only 3rd-order accuracy even in the very smooth regions. Another is to use a lot of logical statements which affect the convergence rate and the efficiency of parallel computing.

Recently, G.-S. Jiang and C.-W. Shu developed a 5th-order weighted ENO scheme [2] based on the third-order accurate difference scheme in the flux form. We found that the third-order accurate ENO scheme given in [2] with r=3 is the same as the ENN scheme without the limiters. Naturally, the ENN scheme would be expanded to 5th-order accurate scheme by using the idea of deriving the 5th-order WENO difference scheme.

In this paper, we have constructed the higher accuracy difference scheme based on the ENN scheme. The new scheme has 5th-order accuracy in smooth regions and can keep the essencially non-oscillatory property.

We tested the new scheme's accuracy by using a linear initial problem and tested its non-oscillatory property by using a nonlinear initial problem. At last, we applied the new scheme to compute the problem of shock-boundary-layer interaction. Numerical results showed that the new scheme is efficient.

2. ENN Scheme and Several High Order Accuracy Central Schemes

Consider a scalar conservative hyperbolic equation

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0 \tag{1}$$

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where f is a flux function and can be splitted into two parts, i.e. $f(u)=f^{+}(u)+f^{-}(u)$ and $df^{+}(u)/du \geq 0$ *and* $df^{-}(u)/du \leq 0$. In this paper we define $f^{\pm}(u)=\frac{1}{2}(f(u)\pm\alpha u)$ and $\alpha=max|f'(u)|$ for one-dimensional equation. The semi-discrete conservative difference scheme can be written as follows

$$\frac{du_j}{dt}+\frac{(h_{j+\frac{1}{2}}-h_{j-\frac{1}{2}})}{\Delta x}=0 \tag{2}$$

where the numerical flux $h_{j+\frac{1}{2}}=h^{+}_{j+\frac{1}{2}}+h^{-}_{j+\frac{1}{2}}$.

(1) The ENN scheme[1]

$$h^{+}_{j+\frac{1}{2}}=\begin{cases} f^{+}_{j}+\frac{1}{2}\Delta f^{+}_{j+\frac{1}{2}}-\frac{1}{6}ms(D^{+}_{j},D^{+}_{j+1}) & if \mid \Delta f^{+}_{j+\frac{1}{2}} \mid\leq\mid \Delta f^{+}_{j-\frac{1}{2}} \mid \quad (3) \\ f^{+}_{j}+\frac{1}{2}\Delta f^{+}_{j-\frac{1}{2}}+\frac{1}{3}ms(D^{+}_{j},D^{+}_{j-1}) & if \mid \Delta f^{+}_{j+\frac{1}{2}} \mid>\mid \Delta f^{+}_{j-\frac{1}{2}} \mid \quad (4) \end{cases}$$

$$h^{-}_{j+\frac{1}{2}}=\begin{cases} f^{-}_{j+1}-\frac{1}{2}\Delta f^{-}_{j+\frac{3}{2}}+\frac{1}{3}ms(D^{-}_{j+1},D^{-}_{j+2}) & if \mid \Delta f^{-}_{j+\frac{3}{2}} \mid\leq\mid \Delta f^{-}_{j+\frac{1}{2}} \mid \quad (5) \\ f^{-}_{j+1}-\frac{1}{2}\Delta f^{-}_{j+\frac{1}{2}}-\frac{1}{6}ms(D^{-}_{j},D^{-}_{j+1}) & if \mid \Delta f^{-}_{j+\frac{3}{2}} \mid>\mid \Delta f^{-}_{j+\frac{1}{2}} \mid \quad (6) \end{cases}$$

where $D_j=\Delta f_{j+\frac{1}{2}}-\Delta f_{j-\frac{1}{2}}$ and the ms(a,b) is defined below

$$ms(a,b)=\begin{cases} a & \mid a \mid\leq\mid b \mid \\ b & \mid a \mid>\mid b \mid \end{cases} \tag{7}$$

(2) 4th-order accurate central schemes [3]

$$h^{+}_{j+\frac{1}{2}}=f^{+}_{j}+\frac{1}{2}\Delta f^{+}_{j+\frac{1}{2}}-\frac{1}{12}D^{+}_{j}-\frac{1}{12}D^{+}_{j+1} \tag{8}$$

$$h^{+}_{j+\frac{1}{2}}=f^{+}_{j}+\frac{1}{2}\Delta f^{+}_{j-\frac{1}{2}}+\frac{1}{4}D^{+}_{j}+\frac{1}{12}D^{+}_{j-1} \tag{9}$$

$$h^{-}_{j+\frac{1}{2}}=f^{-}_{j+1}-\frac{1}{2}\Delta f^{-}_{j+\frac{3}{2}}+\frac{1}{4}D^{-}_{j+1}+\frac{1}{12}D^{-}_{j+2} \tag{10}$$

$$h^{-}_{j+\frac{1}{2}}=f^{-}_{j+1}-\frac{1}{2}\Delta f^{-}_{j+\frac{1}{2}}-\frac{1}{12}D^{-}_{j+1}-\frac{1}{12}D^{-}_{j} \tag{11}$$

(3) 5-order accurate central scheme

$$h^{+}_{j+\frac{1}{2}}=f^{+}_{j}+\frac{3}{5}\times(\frac{1}{2}\Delta f^{+}_{j+\frac{1}{2}}-\frac{1}{12}D^{+}_{j}-\frac{1}{12}D^{+}_{j+1})+\frac{2}{5}\times(\frac{1}{2}\Delta f^{+}_{j-\frac{1}{2}}+\frac{1}{4}D^{+}_{j}+\frac{1}{12}D^{+}_{j-1}) \tag{12}$$

$$h^{-}_{j+\frac{1}{2}}=f^{-}_{j+1}+\frac{2}{5}\times(-\frac{1}{2}\Delta f^{-}_{j+\frac{3}{2}}+\frac{1}{4}D^{-}_{j+1}+\frac{1}{12}D^{-}_{j+2})+\frac{3}{5}\times(-\frac{1}{2}\Delta f^{-}_{j+\frac{1}{2}}-\frac{1}{12}D^{-}_{j+1}-\frac{1}{12}D^{-}_{j}) \tag{13}$$

3. Numerical Method

For simplicity, we show only the positive part of the splitted flux, and the negative part of the splitted flux are symmetric with respect to $x_{j+\frac{1}{2}}$.

From above equations, it can be seen that the combinal coefficients in (8) from (3) are

$$C^1_1=\frac{1}{2},C^1_2=\frac{1}{2};$$

the combinal coefficients in (9) from (4) are

$$C^2_1=\frac{3}{4},C^2_2=\frac{1}{4};$$