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A ROBUST TRUST REGION ALGORITHM FOR SOLVING GENERAL NONLINEAR PROGRAMMING[*1)]

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Abstract

The trust region approach has been extended to solving nonlinear constrained optimization. Most of these extensions consider only equality constraints and require strong global regularity assumptions. In this paper, a trust region algorithm for solving general nonlinear programming is presented, which solves an unconstrained piecewise quadratic trust region subproblem and a quadratic programming trust region subproblem at each iteration. A new technique for updating the penalty parameter is introduced. Under very mild conditions, the global convergence results are proved. Some local convergence results are also proved. Preliminary numerical results are also reported.

Key words: Trust region algorithm, Nonlinear programming.

1. Introduction

Trust region methods are iterative. As a strategy of globalization, the trust region approach was introduced into solving unconstrained optimization and proved to be efficient and robust. An excellent survey was given by Moré(1983). The associated research with trust region methods for unconstrained optimization can be found in Fletcher(1980), Powell(1975), Sorensen(1981), Shultz, Schnabel and Byrd(1985), Yuan(1985). The solution of the trust region subproblem is still an active studying area, see Stern and Wolkowicz(1994), Peng and Yuan(1997) et al.

Since the 80's the trust region approach has been extended to solving nonlinear constrained optimization. Most of these extensions consider only equality constraints, and the global convergence theories are based on strong global regularity assumptions, for example, see Byrd, Schnabel and Shultz(1987), Vardi(1985), Omojokun(1989), Powell and Yuan(1991), Dennis, El-Alem and Maciel(1997), Dennis and Vicente(1997). At each iteration of an algorithm given by Omojokun(1989), the trial step consists of a normal direction step and a null space step. Similarly, Dennis, El-Alem and Maciel(1997) considered the method which replaced the normal component by a quasi-normal direction and developed its global convergence theory. Dennis and Vicente(1997) proved that under suitable conditions their method will converge to the second-order optimal point. For general constrained optimization, Fletcher(1981) proposed a trust region method which is based on the L_1 nonsmooth exact penalty function. Burke and

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Han(1989), Liu and Yuan(1998) have extented Fletcher's approach to other penalty functions. Burke(1992) presented a general framework for trust region algorithms for constrained problems. Without requiring any regularity assumption, Burke proved that his method converges to the points which satisfies certain first-order optimality conditions. Similar to Fletcher(1981) and Burke(1992), Yuan(1995) proposed a new trust region algorithm for solving the optimization with equality and inequality constraints. Under mild conditions, Yuan(1995) proved the global convergence of his algorithm and established local convergence results.

In this paper, we consider the general nonlinear programming problem

$$\min \quad f(x) \tag{1.1}$$

$$s.t. \quad c_i(x) = 0, \quad i \in E, \tag{1.2}$$

$$c_i(x) \geq 0, \quad i \in I, \tag{1.3}$$

where $f(x)$, $c_i(x)(i \in E \cup I)$ are real valued continuously differentiable functions on $\Re^n$, $E = \{1, 2, \cdots, m_e\}$ and $I = \{m_e+1, \cdots, m\}$ are two index sets with the integers m_e and m satisfying $m \geq m_e \geq 0$. If $m_e = m > 0$, (1.1)-(1.3) is the optimization with only equality constraints.

Successive quadratic programming (SQP) methods are very efficient for solving problem (1.1)–(1.3), see Han(1977), Powell(1978), Burke and Han(1989), Burke(1989). At each iteration, the original SQP method, developed by Wilson, Han and Powell, generates a new approximate to the solution by the procedure

$$x^+ = x + sd, \tag{1.4}$$

where x is the current point, d is a search direction which minimizes a quadratic model subject to linearized constraints and s is the steplength along the direction and is decided by some line search procedure. Under certain conditions, SQP methods converge superlinearly. The requisite consistency of the linearized constraints of the QP subproblem, however, is its serious limitation. In order to handle the inconsistency of the linearized constraints, Liu and Yuan(1998) presented a modified SQP algorithm which solves an unconstrained piecewise quadratic subproblem and a quadratic programming subproblem at each iteration. The algorithm is a natural extension of the original SQP method since it solves the same subproblems as the original SQP method at the feasible points of the original problem, and it coincides with the original method when the iterates are sufficiently close to the solution. Moreover, in order to ensure the fast rate convergence, it seems reasonable to use the second-order information to generate the normal direction, instead of using the first-order term only (for example, see Burke(1989) and Burke and Han(1989)). For optimization with only equality constraints, the normal direction and the null space direction are independent, so the search direction can be computed parallelly (see Liu(1998)).

In this paper, we present a new trust region algorithm for problem (1.1)-(1.3). The new algorithm is based on the SQP method of Liu and Yuan(1998). The trial step is computed by solving an unconstrained piecewise quadratic trust region subproblem and a quadratic programming trust region subproblem at each iteration. A motivation for using trust region techniques is that trust region approach is robust and it can applied to ill-conditioned problems. Our algorithm is similar to Burke(1992) and Yuan(1995), but there remain fundmental differences. For equality constrained case, our method is also similar to the null space and range space approach analyzed by Dennis, El-Alem and Maciel(1997) and Dennis and Vicente(1997). A new technique for updating the penalty parameter is introduced. Under very mild conditions, the global convergence results are proved. Local superlinear convergence results are also proved. Preliminary numerical results are also reported.

This paper is organized as follows. In section 2 we present our algorithm. Some global convergence results of our algorithm are proved in section 3. The local analyses are given in section 4. In section 5, we report some preliminary numerical results.