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A SPLINE METHOD FOR SOLVING TWO-DIMENSIONAL FREDHOLM INTEGRAL EQUATION OF SECOND KIND WITH THE HYPERSINGULAR KERNEL*[1)]

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Abstract

The purpose of this paper is to adopt the quasi-interpolating operators in multivariate spline space $S_2^1(\Delta_{mn}^{2*})$ to solve two-dimensional Fredholm Integral Equations of second kind with the hypersingular kernels. The quasi-interpolating operators are put forward in ([7]). Based on the approximation properties of the operators, we obtain the uniform convergence of the approximate solution sequence on the Second Kind Fredholm intergral equation with the Cauchy singular kernel function.

Key words: Hypersingular integral, Finite-part integral, Quasi-interpolating operator, Nonuniform type-2 triangulation.

1. Introduction

In recent years, the boundary element methods became a reliable and powerful numerical methods for solving the boundary value problems, such as elastoplasticity, etc. In these methods, the original problem is reduced to a boundary integral equation. For the one dimensional boundary, a lot of methods have been put forward recently. But for the two dimensional boundary situation, it is not so easy to be done because the partition can be very complicated. Since P. Zwart obtained an expression of bivariate B-spline [2], R-H Wang and C.K. Chui obtained a quasi-interpolating operators of $S_2^1(\Delta_{mn}^2)$ on uniform type-2 triangulation and its approximation properties ([1]) which have widespread applications in Mechanics and Engineering. Furthermore, R-H Wang and C.K. Chui also obtained the function with minimum support in $S_2^1(\Delta_{mn}^{2*})$ on non-uniform type-2 triangulation and the basis of $S_2^1(\Delta_{mn}^{2*})$ ([4]). In ([7]), we introduced some quasi-interpolating operators of $S_2^1(\Delta_{mn}^{2*})$ on non-uniform type-2 trangulation and show their approximation properties. By using the operators we constructed cubature formulas which can be used to solve hypersingular integrals that arisen from many mechanics and engineering problems.

In present paper, we give a method for solving the two-dimensional Fredholm Integral Equation with hypersingular kernel based on quasi-interpolating operators in $S_2^1(\Delta_{mn}^{2*})$ and prove the uniform convergence of the approximate solution sequence.

2. Quasi-Interpolating Operators of $S(\Delta_{mn}^{2*})$

Let Δ_{mn}^{2*} be a non-uniform type-2 triangulation on the domain $\Omega : [a,b] \otimes [c,d]$, and

$$x_{-2} < x_{-1} < a = x_0 < \cdots < x_m = b < x_{m+1} < x_{m+2},$$
$$y_{-2} < y_{-1} < c = y_0 < \cdots < y_n = d < y_{n+1} < y_{n+2}.$$

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First we consider the linear operators

$$V_{mn} : C(\Omega) \to S_2^1(\Delta_{mn}^{2*}); \tag{2.1}$$

$$V_{mn}(f) = \sum_{ij} f\left(\frac{x_i + x_{i+1}}{2}, \frac{y_j + y_{j+1}}{2}\right) B_{ij}(x, y); \tag{2.2}$$

It is similar to the result in ([1]), we have the following results.

Theorem 2.1. *For $f \in P_1$ and $f = xy$, we have $V_{mn}(f) = f$,* (2.3)

Theorem 2.2. *$W_{mn}(f) = f$ for any $f \in P_2$.*

In terms of theory 2.1, 2.2, and [7], we have the following results.

Let

$$\omega_k(f, \delta) = \sup\{|f(x, y) - f(u, v)| : (x, y), (u, v) \in K, |(x, y) - (u, v)| < \delta\}. \tag{2.4}$$

$$\begin{aligned} \delta_{mn} &= \max[h_i, k_j], \\ \delta_{mn}^* &= \max(\sqrt{9h^2 + k^2}, \sqrt{9k^2 + h^2}), \\ h &= \max_i(h_i), k = \max_j(k_j); \end{aligned} \tag{2.5}$$

and where K be a compact set and $K \subset \Omega$.

Theorem 2.3. *Let $f \in C(K)$ and $m, n \geq N_0$. we have*

$$\|f - V_{mn}(f)\|_\Omega \leq \omega_k(f, \delta_{mn}^*), \tag{2.6}$$

if $f \in C^1(K)$ then

$$\|f - V_{mn}(f)\|_\Omega \leq \delta_{mn} \max(\omega_\Omega(f_1, \delta_{mn}/2), (\omega_\Omega(f_2, \delta_{mn}/2), \tag{2.7}$$

if $f \in C^2(K)$ then

$$\|f - V_{mn}(f)\|_\Omega \leq \delta_{mn}^2 \|D^2 f\|. \tag{2.8}$$

Theorem 2.4. *Let $f \in C^2(\Omega)$ and $m, n \geq N_0$. If $f \in C^2(K)$, then*

$$\|f - W_{mn}(f)\|_\Omega \leq \frac{1}{2}\delta_{mn}^2 \max[\omega_\Omega(f_{11}, \delta_{mn}/2), 2\omega_\Omega(f_{12}, \delta_{mn}/2), \omega_\Omega(f_{22}, \delta_{mn}/2)] \tag{2.9}$$

if $f \in C^3(K)$ then

$$\|f - W_{mn}(f)\|_\Omega \leq \frac{1}{12}\delta_{mn}^3 \|D^3 f\|. \tag{2.10}$$

Taking note of $\|W_{mn}\| = 3$. it is easy to prove the theorem in term of Taylor expansion.

3. Cubature Formulas

Here we consider integrals of the form

$$\int_\Omega K_p(v_0; v)\Phi(v) dv, v_0 \in \Omega \subset R^2, \tag{3.1}$$

where the kernel K_p admit the expansion

$$K_p(v_0; v) = \sum_{l=0}^{p} \frac{f_{p-l}(v_0; \theta)}{r^{p+2-l}} + K_p^*(v_0, v). \tag{3.2}$$