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SUPERCONVERGENCE ANALYSIS FOR CUBIC TRIANGULAR ELEMENT OF THE FINITE ELEMENT*[1)]

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Abstract

In this paper, we construct a projection interpolation for cubic triangular element by using othogonal expansion triangular method. We show two fundamental formulas of estimation on a special partion and obtain a superconvergence result of $1-\epsilon$ order higher for the placement function and its tangential derivative on the third order Lobatto points and Gauss points on each edge of triangular element.

Key words: Finite element, Superconvergence, Projection interpolation.

1. Introduction

Although we had proved the superconvergence of quadratic triangular elements before 1985, the superconvergence research of $k(k \geq 3)$-degree triangular elements only has a few advances, e.g.,.Lin , Yan and Zhou (see [15]) prove that the three degree Hermite elements possess superconvergence and Wahlbin (see [5-7]) obtains a rough result by using a fine interior estimation, that is, the placement function or its gradient may have weak superconvergence in the local symmetric points,e.g. the middle points of each edge of a element. In 1989, we have pointed out that Li Bo's example in [8] can be explained that it is difficult to show the superconvergence of higher degree element by the traditional interpolation expansion (see [10]), but it did not show that there is no superconvergence for the higher degree element and we confirm that there is superconvergence for $k(k \geq 3)$-degree triangular elements. In this paper, using orthogonal expansion of triangular element, we construct an projection interpolation for cubic triangular element. After two fundamental formulas of estimation on a special partion are shown, some superconvergence results of the placement functions and their tangential derivatives at the third order Lobatto points and Gauss points on each edge of triangular elements are proved. Moreover we will study the superconvergence of the complicated problem of the finite element method.

For the sake of convenience,we consider the model problem: Find $u \in H_0^1(\Omega)$ such that

$$a(u,v) = (f,v), \ \ \forall v \in H_0^1(\Omega),$$

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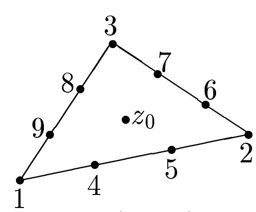
where $a(u,v)=\int_\Omega \nabla u\nabla v dxdy$, $(f,v)=\int_\Omega fvdxdy$, Ω is a smooth or convex polygonal domain . We suppose that Ω is divided into uniform triangles (or local uniform triangles if we consider local superconvergence).Let J^h denote the triangulation, S^h denote the piecewise cubic finite element space over the triangulation J^h. For each triangular element $e\in J^h, e=\triangle z_1z_2z_3$, let $s_i(i=1,2,3)$ denote the opposite side of the vertex z_i of e. h_i is the lenth of the edge s_i $(s_{i+3}=s_i)$, h is the maximum of all lenthes of edges.Let s_i and n_i denote the direction and nomal direction of the edge, respectively and the corresponding directional derivatives are denoted by ∂_i and ∂_{n_i}. $P_k(e)$ and λ_i denote all the k-degree polynomials and area coodinates on element e respectively. $z_i(i=1,\cdots,9)$ are the points of trisection of three edges of element e and z_0 is the barycenter of e (see Figure 1). For all z_i, we may construct basis function $\phi_i\in P_3(e)$ such that $\phi_i(z_j)=\delta_{ij}$.



(Fig.1)

2. Projection Interpolation

Now we construct a new type of interpolation, i.e. projection interpolation like Lin and Zhu in [4] in 1994. It is constructed by projection of a function on polynomial space according to some fashion. Consider firstly the problem of one dimension. Let

$$s=(x_0-h,x_0+h)=(\alpha,\beta).$$

We construct following complete normalizing orthogonal system of polynomials $\{L_n(x)\}$ in the sense of inner product of $L^2(s)$:

$$\begin{array}{l}
L_0(x)=\sqrt{\frac{1}{2}}h^{-1/2}\\
L_1(x)=\sqrt{\frac{3}{2}}h^{-3/2}(x-x_0)\\
L_2(x)=\sqrt{\frac{5}{2}}h^{-5/2}[3(x-x_0)^2-h^2]\\
\cdots\cdots\cdots\cdots\\
L_i(x)=\sigma_i(\frac{d}{dx})^i[A(x)]^i,\ (i\geq 1),\\
\sigma_i=\sqrt{\frac{(2i+1)}{2}}\frac{1}{i!}h^{-i-1/2}=O(h^{-i-1/2})\\
\cdots\cdots\cdots\cdots
\end{array}\tag{2.1}$$