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TWO ITERATION METHODS FOR SOLVING LINEAR ALGEBRAIC SYSTEMS WITH LOW ORDER MATRIX A AND HIGH ORDER MATRIX B: $Y = (A \otimes B)Y + \Phi^{*1)}$

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Abstract

This paper presents optimum an one-parameter iteration (OOPI) method and a multi-parameter iteration direct (MPID) method for efficiently solving linear algebraic systems with low order matrix A and high order matrix B: $Y = (A \otimes B)Y + \Phi$. On parallel computers (also on serial computer) the former will be efficient, even very efficient under certain conditions, the latter will be universally very efficient.

Key words: System of algebraic equations, Iteration method, Iteration direct method, Solution method for stiff ODEs

1. Introduction

It is well known that for IVP of stiff ODEs

$$y' = f(y), \quad t_0 < t \leq T \quad y(t_0) = y_0 \in R^m, \quad f : \Omega \in R^m \in R^m, \quad m \gg 0 \tag{1.1}$$

implicit method with good stability have to be used, e.g., IRK methods[7], implicit block methods[4,12–17,18], etc. At each integral step, each of all these methods brings about solving block nonlinear equation systems

$$Y = h(A \otimes I_m)F(Y) + \Phi_1, \quad A \in R^{s*s}, \quad Y, F(Y), \Phi \in R^{ms}, \quad ms \gg 0, \tag{1.2}$$

where h is the stepsize, $\otimes$ kronecker product, $I_m \in R^m$ identity matrix, $Y = (y_1^T, y_2^T, \cdots, y_s^T)^T$, $F(Y) = (f(y_1)^T, \cdots, f(y_s)^T)^T$. Now, efficiently solving (1.2) become a key of efficiently solving (1.1).

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Let

$$f(y) = \overline{B}y + g, \tag{1.3}$$

where $\overline{B} \in R^{m*m}$ is a constant matrix, $g \in R^m$ is a constant vector. For the definite problems of linear evolution equations systems

$$\frac{\partial u}{\partial t} = Lu + l(t, x_1, x_2, \cdots, x_q),$$

where L is a linear partial differential operator with respect to the space variables $x_1, x_2, \cdots, x_q$,
$l(t, x_1, x_2, \cdots, x_q)$ is a known continuous function of the time variable t and space variable $x_1, x_2, \cdots, x_q$. Using the semi-discrete method, we can obtain (1.1)(1.3). Under the condition (1.3), (1.2) can be written as a linear equation system

$$Y = (A \otimes B)Y + \Phi, \tag{1.4}$$

here $B = h\overline{B}, \quad \Phi = h((Ae) \otimes g) + \Phi_1, e = (1, 1, \cdots, 1)^T \in R^s$.

The research of solution method for (1.2) have had a number of results[2,3,5,6,17]. We attempt to set up an universal efficient solution method for (1.1), (1.2) by the way of the construction of efficient solution method for (1.4). This aim have been achieved. As the space is limited, the paper only discusses solution methods for (1.4). As to solution methods for (1.2), we shall discuss then in another paper.

In order to set up an universal efficient solution method for (1.1)(1.3)(1.4) which can be generalized to establish an universal efficient solution method for (1.1)(1.2), we do some analyses for (1.4) produced from (1.2)(1.3).

Unlike general linear systems, (1.4) produced from (1.1)(1.3) have following features:

i) A in (1.4) is only determined by the method used by solution of (1.1)(1.3), its orders is lower. Usually, $s \in [2, 6]$, about at most doesn't exceed 10;

ii) To ensure the accuracy of numerical solution, the discrete stepsize h_i adopted in the directions of the space variable $x_i, i = 1(1)q$ are sufficient small, therefore m is a large number. When $m \gg 0$, to solve (1.4) need to use parallel computers(or vector computer) usually.

iii) For the accurate solution $Y = Y^*$ of (1.4), there is an initial approximation Y_0 with good accuracy.

Establishing an efficient solution method for (1.1)(1.2) (or ((1.1)(1.3)(1.4)) we must consider all of the three points.

For a matrix equation

$$A_1X + XB_1 = C_1, \quad A_1 \in R^{s*s}, \quad B_1 \in R^{m*m}, \quad C_1, \quad X \in R^{s*m}, \tag{1.5}$$

which is equivalent to

$$\overline{X} = -(A_1^{-1} \otimes B_1^T)\overline{X} + (A_1^{-1} \otimes I_m)\overline{C}_1$$