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PARALLEL MULTI-STAGE & MULTI-STEP METHOD IN ODES*

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Abstract

In this paper, the theory of parallel multi-stage & multi-step method is discussed, which is a form of combining Runge-Kutta method with linear multi-step method that can be used for parallel computation.

Key words: Ordinary differential equations, Parallel simulation.

1. Introduction

Early in 1966, W.L.Miranker and W.Liniger [3] gave a kind of parallel Runge-Kutta method of order 2 and order 3. But unfortunately, when the method given in [3] was applied to test equation $y' = \lambda y$, $\mathrm{Re}\lambda < 0$, it's numerically unstable. In that time, W.L.Miranker and W.Liniger then expressed their expectation for a parallel Runge-Kutta method with numerical stability. At the beginning of 1990's, some kinds of parallel Runge-Kutta method were considered in [1] and [2] with some modification of those formulae given in [3], and their absolute stability regions were discussed. Recently, the theory of combination method is set up, and some formulae of this method are given in [4], which combines Runge-Kutta method with linear multi-step method. This combination method can be parallel computed just like the parallel Runge-Kutta method. In this paper, the theory of parallel multi-stage & multi-step method is discussed, which is another form of combining Runge-Kutta method with linear multi-step method.

Some algorithms based on this method are part of PASL (Parallel Algorithm Software Library) on S10 parallel computer (MIMD) which was made by Aero-Space Department of China in 1991.

2. Basic Theory

Consider the system of differential equations

$$y' = f(y), \quad y(x_0) = y_0, \quad f : \mathbf{R}^m \to \mathbf{R}^m \tag{2.1}$$

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parallel r-stage & r-step method is constructed with the following form

$$y_{n+1} = BY_n + hCU_nD \tag{2.2}$$

where

$$B = (b_1, b_2, ..., b_r), \qquad C = (1, c_2, ..., c_r)$$

$$D = (d_1, d_2, ..., d_r)^T, \qquad Y_n = (y_n, y_{n-1}, ..., y_{n-r+1})^T$$

$$U_n = \begin{pmatrix} K_{1,n} & K_{1,n-1} & \cdots & K_{1,n-r+1} \\ 0 & K_{2,n-1} & \cdots & K_{2,n-r+1} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & K_{r,n-r+1} \end{pmatrix}$$

$$K_{1,n} = f(y_n), K_{1,i} = f(y_i), K_{j,i} = f(\sum_{l=1}^{j} w_{j,l} y_{i+l-1} + h(\sum_{l=1}^{j-1} \beta_{j,l} K_{l,i})), \sum_{l=1}^{j} w_{j,l} = 1$$

$$j = 2, 3, ..., n-i+1, \quad i = n-1, n-2, ..., n-r+1.$$

We can find out the method (2.2) can be executed in parallel by r-processes, that $K_{1,n}$, $K_{2,n-1}, ..., K_{r,n-r+1}$ can be computed synchronouslly.

Theorem 2.1. *If the following conditions are satisfied*

(i) *the module of roots of*

$$\lambda^r - B(\lambda^{r-1}, \lambda^{r-2}, ..., \lambda^0)^T = 0, \quad \lambda \epsilon \mathbf{C}$$

are no more than 1 and the roots with module 1 are single

(ii) *for* $y_i = y(x_i), \quad i = n, n-1, ..., n-r+1$, *have*

$$y(x_{n+1}) - BY_n - hCU_nD = O(h^{m+1})$$

then (2.2) is convergent with order m.

Proof. Let

$$\bar{B} = \begin{pmatrix} b_1 & b_2 & \cdots & b_r \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1 & 0 \end{pmatrix}, \bar{C} = \begin{pmatrix} 1 & c_2 & \cdots & c_r \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

so we have

$$Y_{n+1} = \bar{B}Y_n + h\bar{C}U_nD$$

$$Y(x_{n+1}) = \bar{B}Y(x_n) + h\bar{C}U(x_n)D + O(h^{m+1})$$

Let

$$e_n = y(x_n) - y_n$$

$$E_n = (e_n, e_{n-1}, \cdots, e_{n-r+1})^T$$