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# SUBSPACE SEARCH METHOD FOR A CLASS OF LEAST SQUARES PROBLEM[*1)]

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**Abstract**

A subspace search method for solving a class of least squares problem is presented in the paper. The original problem is divided into many independent subproblems, and a search direction is obtained by solving each of the subproblems, as well as a new iterative point is determined by choosing a suitable steplength such that the value of residual norm is decreasing. The convergence result is also given. The numerical test is also shown for a special problem.

*Key words*: Subspace search method, A class of least squqres problem, Convergence analysis.

## 1. Introduction

Now the least squares problem is considered as follows:

$$Min\ r(x,y) = \frac{1}{2}\|Ax + By - b\|^2 \quad s.t. \quad x \ \geq 0 \tag{1.1}$$

where $A \in R^{m\times t}$, $B \in R^{m\times q}$, and $b \in R^m$ are given constant matrices and vectors, respectively.

These problems arise in many areas of applications, such as scientific and engineering computing, physics, statistics, fitted curve, economic, mathematical programming, social science, and as a component part of some large computation problem, as an example, a nonlinear least squares problem is approximated locally by using of various linearization schemes.

This problem often is to analyze and solve a systems of linear algebraic equations, which may be overdetermined, underdetermined , or exactly determined, and may or may not be consistent with linear and inequality constraints. Many successful algorithms for solving this problems have been studied during past decades, and the decomposition method (QR) is a popular approach to use for solving the problems. G.H.Golub has contributed many significant ideas and algorithms relating to this problems in both theoretical and practice, and a practical and useful numerical methods for solving this problems were stated in [2] in details. A different techniques have also developed for the problems.

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In this paper, we present a subspace search method for solving the problem (1.1). The main steps of the algorithm are to divide the problem (1.1) into independent subproblems at an initial feasible point and solve each of these subproblems to obtain a search direction, and then to determine a new feasible iterative point by choosing a suitable steplength such that the value of the residual norm is decreasing. the convergence of the algorithm is proved under certain assumptions. The main feature of the algorithm is that large scale problom (1.1) can be transformed into many small independent subproblems, and all the subproblems can be solved simultaneously.

This paper is organized as follows. in section 2 we describe the algorithm. The convergence results are proved under certain assumptions in section 3, A modification algorithm and numerical test is also given in section 4.

## 2. Derivation of the Algorithm

This section deals with the basic ideas that how to construct the subspace serach algorithm for solving the problem (1.1). Without loss of generality, assume that vector $x \in R^t$ and $y \in R^q$ can be divided into $(x_1^T, x_2^T, \cdots, x_{l_1}^T)^T$ and $(y_1^T, y_2^T, \cdots, y_{l_2}^T)^T$, and $x_i \in R^{n_i}$, $y_j \in R^{n_j}$, respectively, and that $\sum_{i=1}^{l_1} n_i = t, \sum_{j=1}^{l_2} n_j = q$. Accordingly, matrix $A$ and $B$ can be also divided into following form $A = (A_1, A_2, \cdots, A_{l_1}) \quad B = (B_1, B_2, \cdots, B_{l_2})$ , where $A_i \in R^{m\times n_i}$ $(i = 1, 2, \cdots, l_1)$, $B_j \in R^{m\times n_j}$ $(j = 1, 2, \cdots, l_2)$. Therefore, the function $r(x, y)$ can be expressed as following form

$$r(x,y) = \frac{1}{2}\|\sum_{i=1}^{l_1} A_i x_i + \sum_{j=1}^{l_2} B_j y_j - b\|^2 \tag{2.1}$$

Now we analyze the properties of function $r(x, y)$ in the neighbourhood of a given initial vector $(\bar{x}, \bar{y})$. Assume that $(x, y)$ is in the neighbourhood of $(\bar{x}, \bar{y})$ and let

$$x = \bar{x} + (x - \bar{x}) \quad y = \bar{y} + (y - \bar{y}) \tag{2.2}$$

Substituting (2.2) into (2.1),and it is easy to derive that

$$\begin{aligned} r(x,y) = \frac{1}{2}[\sum_{i=1}^{l_1} \|\bar{r} - A_i(x_i - \bar{x}_i)\|^2 + \sum_{j=1}^{l_2} \|\bar{r} - B_j(y_j - \bar{y}_j)\|^2 - (l_1 + l_2 - 1)\bar{r}] + \\ \frac{1}{2}\hat{r}(x,y) = \sum_{i=1}^{l_1} \xi_i(x_i, \bar{x}, \bar{y}) + \sum_{j=1}^{l_2} \zeta_j(y_j, \bar{x}, \bar{y}) - \frac{l_1 + l_2 - 1}{2}\bar{r} + \frac{1}{2}\hat{r}(x,y) \end{aligned} \tag{2.3}$$

where $\bar{r} = r(\bar{x}, \bar{y})$, and $\hat{r}(x, y)$ is an error of order $O(\|x - \bar{x}\|^2 + \|y - \bar{y}\|^2)$. Namely,

$$\begin{aligned} \hat{r}(x,y) = \sum_{i=1}^{l_1}\sum_{j\neq i}^{l_1}(x_i - \bar{x}_i)A_i^T A_j(x_j - \bar{x}_j) + \sum_{i=1}^{l_2}\sum_{j\neq i}^{l_2}(y_i - \bar{y}_i)B_i^T B_j(y_j - \bar{y}_j) \\ + 2\sum_{i=1}^{l_1}\sum_{j=1}^{l_2}(x_i - \bar{x}_i)A_i^T B_j(y_j - \bar{y}_j) \end{aligned} \tag{2.4}$$

and

$$\xi_i(x_i, \bar{x}, \bar{y}) = \frac{1}{2}\|\bar{r} - A_i(x_i - \bar{x}_i)\|^2, \qquad i = 1, 2, \cdots, l_1 \tag{2.5}$$