Journal of Computational Mathematics, Vol.18, No.1, 2000, 13–24.

A TRUST REGION-TYPE METHOD FOR SOLVING MONOTONE VARIATIONAL INEQUALITY*[1)]

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Abstract

The Newton method for variational inequality problem is locally and quadratically convergent. By using a differentiable merit function, Taji, Fukushima and Ibaraki[1] have given a globally convergent modified Newton method for the strongly monotone variational inequality problem and proved their method to be quadratically convergent under some additional assumptions. In this paper we propose to present a trust region-type modification of Newton method for the strictly monotone variational inequality problem using the same merit function as that in [1]. It is then shown that our method is well defined and globally convergent and that, under the same assumptions as those in [1], our algorithm reduces to the basic Newton method and hence the rate of convergence is quadratic. Computational experimence indicates the efficiency of the proposed method.

Key words: Variational inequality problem, Trust region method, Global convergence, Quadratic convergence.

1. Introduction

Let S be a nonempty closed convex subset of R^n and let $F : R^n \to R^n$ be a continuous mapping. The variational inequality problem

$$\text{Find } x^* \in S \text{ such that } \langle F(x^*), x - x^* \rangle \geq 0 \text{ for all } x \in S \qquad \text{(VIP)}$$

is widely used to study various equilibrium models arising in economic, operations research, transportation and regional sciences[2,3], where $\langle \cdot, \cdot \rangle$ denotes the inner product in R^n. Many iterative methods for (VIP) have been developed, for example, projection methods[7,8], the nonlinear Jacobi method[5], the successive overrelaxation method[9]

* Received May 24, 1996.
1)Project supported by the National Natural Science Foundation of China.

and generalized gradient method[10,11]. These methods usually converge to a solution of (VIP) under certain conditions on the mapping F and rates of convergence are generally linear[2,5,12].

It is well known that Newton method for nonlinear equations and unconstrained minimization problems converges locally and quadratically. For (VIP), Newton method generates a sequence of iterates $\{x^k\}$, where x^{k+1} is a solution of the linearized variational inequality problem

$$\text{Find } x \in S \text{ such that } \langle F(x^k) + \nabla F(x^k)^T(x - x^k), y - x\rangle \geq 0 \text{ for all } y \in S. \quad (0)$$

It has been shown that[5], under assumptions that x^* is a regular solution of (VIP) and $\nabla F(x)$ is Lipschitz continuous around x^*, the sequence converges quadratically to x^* if the starting point x^0 is sufficiently close to x^*.

Recently, Marcotte and Dussault (1989), Taji, Fukushima and Ibaraki (1993) presented a globally convergent Newton method for (VIP) by incorporating line search strategies. Marcotte and Dussault's method uses the gap function $g(x) = \max\{\langle F(x), x - y\rangle | y \in S\}$ as a merit function. The function g is generally nondifferentiable and achieves its minimum at a solution of (VIP) on S. The set S is assumed to be compact in order that the function g is well-defined. It is shown that when F is monotone, the method is globally convergent when line searches are exact and that under the joint assumptions of strong monotonicity and strict complementarity, the rate of convergence is quadratic. Taji, Fukushima and Ibaraki's method employes a differentiable merit function proposed by Fukushima[6], whose minimizer on S coincides with the solution of (VIP). The method allows inexact line searches and does not rely on the compact assumption of the set S. When F is strongly monotone, the method is globally convergent and, under additional assumptions that the set S is polyhedral convex, $\nabla F(x)$ is locally Lipschitz continuous and strictly complementarity condition holds at the unique solution x^* of (VIP), the rate of convergence is quadratic.

In this paper, we propose a trust region modification of Newton method for (VIP). Fukushima's differentiable mertit function is used. When F is strictly monotone rather than strongly monotone, the proposed algorithm is well-defined and converges globally to the unique solution of (VIP). Under the same assumptions as made in [1], it is shown that for k sufficiently large, no trust region subproblem involves, therefore, the algorithm reduces to the basic Newton method and hence the rate of convergence is also quadratic.

The paper is organized as follows. In section 2, we review some preliminary results of the monotone mapping F and the merit function that are useful in the subsequent sections. In section 3, we present our trust region-type modification of Newton method for solving monotone variational inequality and prove that it is well defined. In section 4, we establish a global convergence theorem without the assumptions that F is strongly monotone and S is compact. The rate of convergence of our algorithm is given in section 5. In section 6, we present some computational results.