Journal of Computational Mathematics, Vol.17, No.6, 1999, 615–628.

ON THE DOMAIN DECOMPOSITION METHOD FOR MORLEY ELEMENT –FROM WEAK OVERLAP TO NONOVERLAP*

Jian-guo Huang
(*Department of Applied Mathematics, Shanghai Jiao Tong University, Shanghai 200240, China*)

Abstract

In this paper, following our original ideas[9], we first consider a weakly overlapping additive Schwarz preconditioner according to the framework of [2] for Morley element and show that its condition number is quasi-optimal; we then analyze in detail the structure of this preconditioner, and after proper choices of the inexact solvers, we obtain a quasi-optimal nonoverlapping domain decomposition preconditioner in the last. Compared with [12], [13], it seems that according to this paper's procedure we can make out more thoroughly the relationship between overlapping and nonoverlapping domain decomposition methods for nonconforming plate elements, and certainly, we have also proposed another formal and simple strategy to construct nonoverlapping domain decomposition preconditioners for nonconforming plate elements.

Key words: Morley element, Domain decomposition, Weak overlap.

1. Introduction

We consider, for simplicity, the following clamped plate bending problem:

$$\begin{cases} \Delta^2 u = f, & (\Omega), \\ u = \partial_n u = 0, & (\partial\Omega), \end{cases} \tag{1.1}$$

where Ω is a plane polygonal domain and n denotes the unit outward normal along the boundary $\partial\Omega$. The related variational form is

$$\begin{cases} u \in V \equiv H_0^2(\Omega), \\ a(u,v) = (f,v), & v \in V, \end{cases} \tag{1.2}$$

where $a(u,v) \equiv \int_\Omega [\Delta u \Delta v + (1-\nu)(2\partial_{12}u\partial_{12}v - \partial_{11}u\partial_{22}v - \partial_{22}u\partial_{11}v)]dx$, $(f,v) \equiv \int_\Omega fvdx$, $\nu \in (0, 0.5)$ is the Poisson ratio. Clearly, the above bilinear form $a(\cdot,\cdot)$ satisfies

* Received September 24, 1996.

the boundedness and coercivity estimates:

$$\begin{cases} |a(v,w)| \le (1+\nu)|v|_{2,\Omega}|w|_{2,\Omega}, & v,w \in H^2(\Omega), \\ a(v,v) \ge (1-\nu)|v|^2_{2,\Omega}, & v \in H^2(\Omega). \end{cases} \tag{1.3}$$

Throughout this paper we adopt the standard conventions for Sobolev norms and semi-norms of a function v defined on an open set G:

$$\|v\|_{m,G} \equiv \Big(\int_G \sum_{|\alpha|\le m} |\partial^\alpha v|^2 dx\Big)^{1/2},$$
$$|v|_{m,G} \equiv \Big(\int_G \sum_{|\alpha|= m} |\partial^\alpha v|^2 dx\Big)^{1/2},$$
$$|v|_{m,\infty,G} \equiv \max_{|\alpha|=m} \|\partial^\alpha v\|_{L^\infty(G)}.$$

We shall also denote the space of polynomials of degree less than or equal to l on G by $P_l(G)$.

Let $\bar\Omega = \cup_{K\in T_h}\bar K$ be a quasi-uniform and regular triangulation of Ω[4], the diameter size of which is denoted by h, here each $K \in T_h$ is an open triangle. On this triangulation we construct the so-called Morley element[4],[11]:

$V^h \equiv \{v : v|_K \in P_2(K)$, v(respectively, $\partial_n v$) is continuous at each vertex p of K(respectively, each edge midpoint m of K), $\forall\ K \in T_h\}$,

$$V_0^h \equiv \{v \in V^h:\ v(p)=0,\ p\in\partial\Omega,\ \partial_n v(m)=0,\ m\in\partial\Omega\}. \tag{1.4}$$

Here and henceforth, p and m (with or without subscript) represent a vertex and an edge midpoint of the elements in T_h respectively. Then, based on (1.4), the discrete problem of (1.2) reads as follows:

$$\begin{cases} a_h(u_h,v_h) = (f,v_h), & v_h \in V_0^h, \\ u_h \in V_0^h, \end{cases} \tag{1.5}$$

where $a_h(v,w) \equiv \sum_{K\in T_h}\int_K[\Delta v\Delta w + (1-\nu)(2\partial_{12}v\partial_{12}w - \partial_{11}v\partial_{22}w - \partial_{22}v\partial_{11}w)]dx.$

It is well-known that the PCG is a proper method to solve (1.5), and the core step is how to design a well-preconditioned and easily invertible in parallel preconditioner, since the condition number of the discrete system (1.5) is $O(h^{-4})$. In [2], S.C.Brenner proposed a two-level additive Schwarz preconditioner for nonconforming plate elements; the main ingredient is the construction of proper intergrid transfer operators which build important bridges among nonconforming elements and their conforming relatives, and thus the difficulty that subspaces are not nested for nonconforming element case was overcome successfully. In [8], J.Gu and X.Hu presented some extension theorems