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FINITE ELEMENT NONLINEAR GALERKIN COUPLING METHOD FOR THE EXTERIOR STEADY NAVIER-STOKES PROBLEM*1)

Yin-nian He Kai-tai Li Fu-hai Gao
(*College of Science, Xi'an Jiaotong University, Xi'an 710049, China*)

Abstract

In this paper we represent a new numerical method for solving the steady Navier-Stokes equations in three dimensional unbounded domain. The method consists in coupling the boundary integral and the finite element nonlinear Galerkin methods. An artificial smooth boundary is intrdouced seperating an interior inhomogeneous region from an exterior one. The Navier-Stokes equations in the exterior region are approximated by the Oseen equations and the approximate solution is represented by an integral equation over the artificial boundary. Moreover, a finite element nonlinear Galerkin method is used to approximate the resulting variational problem. Finally, the existence and error estimates are derived.

Key words: Navier-Stokes equations, Oseen equations, Boundary integral, Finite element, Nonlinear Galerkin method.

1. Introduction

Nonlinear Galerkin methods are multilevel schemes for the dissipative evolution partial differential equations. They correspond to the splittings of the unknown u : $u = y + z$, where the components are of different order of magnitude with respect to a parameter related to the spatial discretization. The numerical procedure consists of introducing an approximate inertial manifold which is a simplified approximation for the small component z. In particular, z is often obtained as a nonlinear functional of y. These methods have mainly been studied in the case of Fourier spectral discretizations (see [1-4]). The Finite elements approximations are considered in [5-8]. However, these works do not apply to the steady exterior Navier-Stokes equations.

Our purpose here is to present a new numerical method for solving the steady exterior Navier-Stokes equations. First, we introduce an artificial smooth boundary Γ_2 separating an unbounded part Ω_2 from a bounded part Ω_1. Then the Navier-Stokes equations in Ω_2 are approximated by the Oseen equations. By use of the Green

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formula, we derive the coupling problem of the Navier-Stokes equations in Ω_1 combining the boundary integral equation over Γ_2. Next, we present the coupling method of the boundary integral method and the finite element nonlinear Galerkin method for solving the coupling problem. Finally, we prove the well-posedness of the approximate problem and analyse the couvergence rate of the approximate solution. Our result show that the finite element nonlinear Galerkin coupling method is superior to the usual finite element Galerkin coupling method presented in the paper [9].

2. Continuous Coupling Problem

Let Ω_0 be a simply connected bounded open set of R^3 with smooth boundary Γ and let Ω denote the complement of $\Omega_0 \cup \Gamma$. The steady Navier-Stokes problem for a fluid occupying Ω consists in finding the velocity vector u of the fluid and its pressure p^* sach that

$$(N-S)\begin{cases} -\nu\Delta u^* + (u^*\cdot D)u^* + \nabla p^* = f & \text{in } \Omega \\ \text{div } u^* = 0 & \text{in } \Omega \\ u^* = \phi & \text{on } \Gamma \\ u^*(x) \to w_0 & \text{as } x \to \infty \end{cases}$$

Here the coefficient $\nu > 0$ is the dynamic viscosity of the fluids, f represents a density vector of external forces and ϕ is the velocity vector of the flow on Γ satisfying the condition $\int_\Gamma \phi \cdot n ds = 0$, where n denotes the unit vector normal to Γ, exterior to Ω, and w_0 is a constant vector. Moreover, we assume that f has a compact support in Ω.

For simplicity, we deal with the homogeneous boundary condition case of $\phi = 0$ in the sequel, but all the results stated here will still hold if the trace ϕ on Ω is any given sufficient smooth function that admits a solenoidel extension (div $u = 0$) in Ω.

For some sufficient large real number R, we introduce an artificial boundary $\Gamma_2 = \{x \in \Omega;\ |x| = R\}$ embedded in Ω, separating an unbounded region Ω_2 from a bounded region Ω_1 such that Ω_1 contains the support of f and $((u - w_0)\cdot\nabla)u$ is sufficiently small in Ω_2. We shall also denete by n the unit vector normal (from Ω_2) to Γ_2.

With above assumptions, we introduce an approximation (u, p) of (u^*, p^*) such that (u, p) satisfies the following coupling problem

$$(N-S')\begin{cases} -\nu\Delta u + (u\cdot\nabla)u + \nabla p = f & \text{in } \Omega_1 \\ \text{div } u = 0 & \text{in } \Omega_1 \\ u|_\Gamma = 0, \sigma(u,p)\cdot n|_{\Gamma_2} = \lambda^+ & \\ -\nu\Delta u + (w_0\cdot\nabla)u + \nabla p = 0 & \text{in } \Omega_2 \\ \text{div } u = 0 & \text{in } \Omega_2 \\ u|_{\Gamma_2} = u^-,\ \lim\limits_{|x|\to\infty} u(x) = w_0 & \end{cases}$$