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SPIRALS IN 2-D GAS DYNAMICS SYSTEMS*1)

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Abstract

In this paper, the phenomena of spirals are numerically presented by MmB scheme [1] for initial value problems of 2-D gas dynamics ($\gamma = 1.4$), which include 2-D Riemann problems and continuous initial value problems. The numerical results are well coincide with on the exact solution in [2] and the conjectures on solution structure in [3] for 2-D isentropic and adiabatic flows. In isentropic flow, for high speed rotation ($v_0/c_0 > \sqrt{2}$), there is a region of vacuum at the origin and for low speed rotation ($v_0/c_0 < \sqrt{2}$), there is no vacuum, and for adiabatic flow, the structure of spirals is also discussed.

Key words: Spiral, MmB scheme, Conservation laws.

1. Preliminaries

(I) Models

Consider the two models: isentropic and adiabatic flows,

(a) 2-D isentropic flow

$$\begin{cases} \rho_t + (\rho u)_x + (\rho v)_y = 0 \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0 \\ (\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0 \end{cases} \tag{1.1}$$

(b) 2-D adiabatic flow

$$\begin{cases} \rho_t + (\rho u)_x + (\rho v)_y = 0 \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0 \\ (\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0 \\ (\rho(e + \frac{u^2+v^2}{2}))_t + (\rho u(h + \frac{u^2+v^2}{2}))_x + (\rho v(h + \frac{u^2+v^2}{2}))_y = 0 \end{cases} \tag{1.2}$$

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$$e = \frac{p}{(\gamma-1)\rho}, \qquad h = e + \frac{p}{\rho}$$

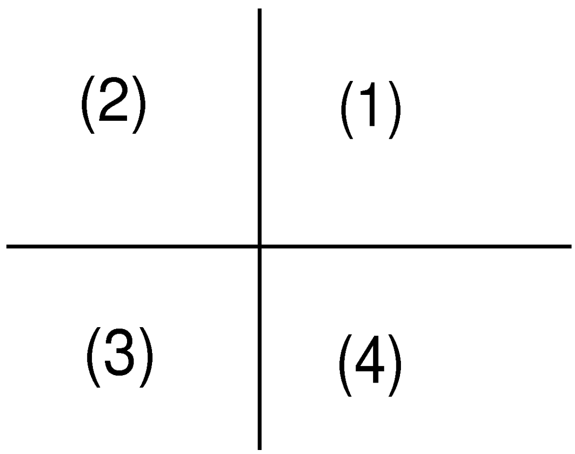
where ρ, (u,v) and p is density, velocity and presure, respectively.
and with the 2-D Riemann data

$$(\rho, u, v)|_{t=0} = (\rho_i, u_i, v_i), \quad (i) = 1, 2, 3, 4 \tag{1.3}$$

or

$$(\rho, p, u, v)|_{t=0} = (\rho_i, p_i, u_i, v_i), \quad (i) = 1, 2, 3, 4 \tag{1.4}$$

where (i)-states are described to



Problem (1.1)(1.3) and (1.2) (1.4) have theoretically studied by characteristic methods [2], and a set of conjectures on the solution structure were presented for the 2-D Riemann problem under the assumption,

Assumption: Each jump in initial data outside the origin projectes exactly one shock wave, rarefaction wave, and slip planes.

The most most interesting conjecture is that there is a spiral in the solution for some Riemann data.

The exact solutions were obtained in the case $\gamma = 2$ for isentropic flow by Zhang and Zheng in [2], the initial data were taken to

$$(u, v, \rho)|_{t=0} = (v_0 \sin\theta, -v_0 \cos\theta, \rho_0)$$

and they got the conclusion that for **high speed rotation** $2c_0^2 < v_0^2$ ($c_0 = \sqrt{p_\rho}$), the solution has region of vacuum at the center; for **low speed rotation** $2c_0^2 > v_0^2$, the solution has no vacuum.

(II) Characteristics and choices of initial data