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CORRECTION METHODS FOR STEADY INCOMPRESSIBLE FLOWS*1)

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Abstract

Correction methods for the steady semi-periodic motion of incompressible fluid are investigated. The idea is similar to the influence matrix to solve the lack of vorticity boundary conditions. For any given boundary condition of the vorticity, the coupled vorticity-stream function formulation is solved. Then solve the governing equations with the correction boundary conditions to improve the solution. These equations are numerically solved by Fourier series truncation and finite difference method. The two numerical techniques are employed to treat the non-linear terms. The first method for small Reynolds number $R = 0 - 50$ has the same results as that in M. Anwar and S.C.R. Dennis' report. The second one for $R > 50$ obtains the reliable results.

Key words: Incompressible flow, vorticity, stream function, numerical solution.

1. Introduction

For semi-periodic incompressible fluid flows, S.C.R. Dennis and co-workers[1-4] solve the vorticity-stream function formulation of the governing equations by the series truncation and finite difference method. Since no boundary condition for the vorticity, they propose the vorticity integral conditions based on Green identity. These methods are effective. But the vorticity integral conditions are implicit. In this paper, the correction method with explicit boundary conditions is proposed. We investigate the steady two-dimensional semi-periodic flow near an infinite array of moving plane walls. This example is developed by M. Anwar and S.C.R. Dennis[3]. They get the numerical solutions by Fourier series and finite-difference approximations. Their series truncation method loses effectiveness for $R > 50$. In the computations by the correction method, we adopt the two numerical techniques to treat the non-linear terms for the various ranges of R. The first method is explicit. The vorticity transport equation with given boundary conditions and the Poisson equation for the stream function with Dirichlet boundary conditions are solved respectively. Then solve a homogeneous problem to correct the solutions. The numerical results for $R = 0 - 50$ are the same as that in [3]. The second method is to solve the coupled vorticity-stream function formulation with

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any given boundary condition of the vorticity. Then again solve the governing equations with the correction boundary conditions to improve the solution. The numerical results for $R > 50$ are reliable. Since the explicit boundary condition of the vorticity, difference equation of the coefficients of Fourier series can be solved by direct method in explicit method. This saves the computational work.

2. Governing Equations

The vorticity-stream function formulation of the steady state incompressible flow is as follows,

$$\begin{cases} \nabla^2\xi = R\left(\frac{\partial\psi}{\partial y}\frac{\partial\xi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\xi}{\partial y}\right), \\ \nabla^2\psi = -\xi, \end{cases} \tag{2.1}$$

where ψ and ξ are the dimensionless stream function and vorticity respectively, R is the Reynolds number.

We consider the example of steady semi-periodic flow as in [3]. The flow is generated by the motion of an infinite array of walls along the y-direction. The velocity components of the moving wall are $u = 0$, $v = -\sin y$, $(-\infty \le y \le \infty)$. Since the flow is periodic and antisymmetrical for y, the boundary conditions are

$$\begin{aligned} &\psi = 0, \frac{\partial\psi}{\partial x} = \sin y, \text{ for } x = 0, \\ &\xi \to 0, \psi \to 0, \text{ as } x \to \infty, \\ &\psi = \xi = 0, \quad \text{for } \ y = 0 \text{ and } y = \pi. \end{aligned}$$

3. Method of Correction Solution

We expand ξ and ψ as Fourier series with respect to y,

$$\begin{cases} \xi(x,y) = \sum\limits_{n=1}^{\infty} g_n(x)\sin ny, \\ \psi(x,y) = \sum\limits_{n=1}^{\infty} f_n(x)\sin ny. \end{cases}$$

By substituting the above series into (2.1), we can get a system of differential equations for Fourier coefficients g_n and f_n,

$$\begin{cases} g_n'' - n^2 g_n = r_n, & n = 1, 2, \cdots, \\ f_n'' - n^2 f_n = -g_n, & n = 1, 2, \cdots, \end{cases} \tag{3.1}$$

where

$$r_n = \frac{R}{2}\sum_{p=1}^{\infty}\{(|n-p|\, f_{|n-p|} - (n+p)f_{n+p})g_p' - p(f_{n+p}' + sgn(n-p)f_{|n-p|}')g_p\},$$

and $sgn(n-p)$ denotes the sign of $(n-p)$, with $sgn(0) = 0$. The boundary conditions in terms of f_n and g_n are $f_n(0) = 0$, $f_n'(0) = \delta_n$, $f_n(\infty) = 0$, $g_n(\infty) = 0$, $n = 1, 2, \cdots$, where $\delta_1 = 1$, $\delta_n = 0$, $n = 2, 3, \cdots$.