Journal of Computational Mathematics, Vol.17, No.1, 1999, 25–32.

LAYER STRIPPING METHOD FOR POTENTIAL INVERSION*1)

Xue-li Dou
(*Beijing Research Institute for Remote Sensing Information, Beijing 100011, China*)
Guan-quan Zhang
(*State Key Laboratory of Scientific and Engineering Computing, ICMSEC, Chinese Academy of Sciences, Beijing 100080, China*)

Abstract

To solve the potential inversion problem of the coupled system for one-way wave equations, the absorbing boundary conditions in the lateral direction are derived. The difference schemes are constructed and a layer stripping method is proposed. Some numerical experiments are presented.

Key words: 2-D potential inversion, Layer stripping method, Absorbing boundary condition

1. Introduction

The potential inversion problem of the following Plasma wave equation is discussed in this paper:

$$\Big[\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial z^2}+v(x,z)\Big]p(x,z,t)=0,\ \ x\in R, z>0, t>0, \tag{1.1}$$

$$p(x,z,0)=\frac{\partial p}{\partial t}(x,z,0)=0, \tag{1.2}$$

$$p(x,0,t)=\delta(t), \tag{1.3}$$

$$\frac{\partial}{\partial z}p(x,0,t)=h(x,t). \tag{1.4}$$

That is, giving an impulse at the surface $z=0$, to determine the wavefield p and potential v from the impulse response h.

There are three kinds of inverse problems of this Plasma wave equation:

(1) To determine the differential equation from its spectral function[1];

(2) To determine the potential from the wave function form at large distance. It is the so-called inverse scattering problem[2,3];

(3) To determine the potential from the response on the boundary to a unit impulse at some reference time $t=0$[4,5].

Our problem belongs to the third kind.

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* Received March 25, 1996.
1)This work was supported by China State Major Key Progect for Basic Research.

In the one dimensional case, there are comprehensive results for this problem. But for the multi-dimensional case, there is not any satisfactory result, whether theoretical or numerical. Because in this case the problem is non-linear and ill-possed.

We have done some theoretical analysis about this potential inversion problem and split the original full-way wave equation into the system of one-way wave equations by using the wave splitting method based on the theory of pseudo-differential operator[6]. In order to make the problem closed, we also transformed the impulse condition (1.3) into the conditions on the characteristic surface by singularity analysis. We also proved the stability of the direct problem for the system of equations, treated as Cauchy problems in the direction of depth.

As the results of wave splitting and singularity analysis the potential inversion problem of the one-way wave equations is [6]:

$$\Big(\frac{\partial}{\partial t}-\frac{\partial}{\partial z}\Big)U_1-\frac{\partial^2}{\partial t^2}\Big[\sum_{m=1}^{n}a_m q_U(s_m)\Big]+\frac{vp}{2}=0, \tag{1.5}$$

$$\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\Big)D_1-\frac{\partial^2}{\partial t^2}\Big[\sum_{m=1}^{n}a_m q_D(s_m)\Big]+\frac{vp}{2}=0, \tag{1.6}$$

$$\frac{\partial p}{\partial z}=U_1-D_1, \tag{1.7}$$

$$U_1=\frac{\partial U}{\partial t}, \tag{1.8}$$

$$D_1=\frac{\partial D}{\partial t}, \tag{1.9}$$

$$\Big(\frac{\partial^2}{\partial t^2}-s_m^2\frac{\partial^2}{\partial x^2}\Big)q_U(s_m,x,z,t)=\frac{\partial^2}{\partial x^2}U(x,z,t), \tag{1.10}$$

$$\Big(\frac{\partial^2}{\partial t^2}-s_m^2\frac{\partial^2}{\partial x^2}\Big)q_D(s_m,x,z,t)=\frac{\partial^2}{\partial x^2}D(x,z,t). \tag{1.11}$$

The initial conditions on the surface $z=0$ are

$$U_1(x,0,t)=-D_1(x,0,t)=\frac{1}{2}h(x,t), \tag{1.12}$$

$$p(x,0,t)=0. \tag{1.13}$$

The conditions for sufficient large value T are:

$$U(x,z,T)=U_1(x,z,T)=q_U(x,z,T)=\frac{\partial q_U}{\partial t}(x,z,T)=0. \tag{1.14}$$

As the results of singularity analysis we get the conditions on the characteristic surface $t=z+0$

$$D(x,z,t=z+0)=-\int_0^z\frac{v(x,y)}{2}dy=g(x,z), \tag{1.15}$$

$$D_1(x,z,t=z+0)=\frac{1}{2}\int_0^z\Big[\frac{\partial^2}{\partial x^2}-v(x,y)\Big]g(x,y)dy-\frac{1}{2}h(x,0), \tag{1.16}$$

$$q_D(x,z,t=z+0)=\frac{\partial q_D}{\partial t}(x,z,t=z+0)=0, \tag{1.17}$$