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LONG TIME ASYMPTOTIC BEHAVIOR OF SOLUTION OF DIFFERENCE SCHEME FOR A SEMILINEAR PARABOLIC EQUATION (II)*1)

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Abstract

In this paper we prove the solution of explicit difference scheme for a semilinear parabolic equation converges to the solution of difference scheme for the relevant nonlinear stationary problem as $t \to \infty$. For nonlinear parabolic problem, we obtain the long time asymptotic behavior of its discrete solution which is analogous to that of its continuous solution. For simplicity, we discuss one-dimensional problem.

Key words: Asymptotic behavior, Explicit difference scheme, Semilinear parabolic equation.

1. Introduction

Let $\Omega = (0,l)$, $f(x) \in H^1(\Omega)$, $u_0(x) \in H^2(\Omega) \cap H_0^1(\Omega)$, $\phi(u) = u^3$, we consider the following initial-boundary value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} - \phi(u) + f(x) \quad \text{in} \quad \Omega \times R_+ \\ u(0,t) = u(l,t) = 0 \\ u(x,0) = u_0(x), \quad x \in \Omega. \end{cases} \tag{1.1}$$

By the usual approach[1–4] we can get the global existence of the solution of (1.1), furthermore, the solution of (1.1) converges to the solution of the following stationary problem (1.2) as $t \to \infty$.

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} - \phi(u) + f(x) = 0 \quad in \quad \Omega \\ u(0,t) = u(l,t) = 0. \end{cases} \tag{1.2}$$

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In [6], [7], the authors considered the explicit scheme for (1.1) as $f(x) = 0$ and only the estimate in L_2 for discrete solution was obtained.

In this paper we prove that the solution of explicit difference scheme for (1.1) converges to the solution of difference scheme for (1.2) as $t \to \infty$.

2. Finite Difference Scheme

The domain Ω is divided into small segments by points $x_j = jh$ $(j = 0, 1, \cdots, J)$, where $Jh = l$, J is an integer and h is the stepsize. Let Δt be time stepsize. For any function $w(x, t)$ we denote the values $w(jh, n\Delta t)$ by w_j^n $(0 \le j \le J,\ n = 0, 1, 2, \cdots)$ and denote the discrete function $w_j^n (0 \le j \le J,\ n = 0, 1, 2, \cdots)$ by w_h^n. We introduce the following notations: $\Delta_+ w_j^n = w_{j+1}^n - w_j^n$ $(0 \le j \le J-1, n = 0, 1, 2, \cdots)$ and $\Delta_- w_j^n = w_j^n - w_{j-1}^n$ $(1 \le j \le J, n = 0, 1, 2, \cdots)$. We denote the discrete function $\dfrac{\Delta_+ w_j^n}{h}$ $(0 \le j \le J-1, n = 0, 1, 2, \cdots)$ by δw_h^n. Similarly, the discrete function $\dfrac{\Delta_+^2 w_j^n}{h^2}$ $(0 \le j \le J-2, n = 0, 1, 2, \cdots)$ is denoted by $\delta^2 w_h^n$.

Denote the scalar product of two discrete functions u_h^n and v_h^m by $(u_h^n, v_h^m) = \sum\limits_{j=0}^{J} u_j^n v_j^m h$.

For $2 \ge k \ge 0$, define discrete norms $\|\delta^k w_h^n\|_p = \Big(\sum\limits_{j=0}^{J-k} \Big| \dfrac{\Delta_+^k w_j^n}{h^k} \Big|^p h \Big)^{\frac{1}{p}}$, $+\infty > p > 1$ and $\|\delta^k w_h^n\|_\infty = \max\limits_{j=0,1,\cdots,J-k} \Big| \dfrac{\Delta_+^k w_j^n}{h^k} \Big|$.

The difference equation associate with (1.1) is:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{\Delta_+ \Delta_- u_j^n}{h^2} - \phi(u_j^n) + f_j \tag{2.1}$$

for $j = 1, \cdots, J-1$ and $n = 1, 2, \cdots\cdots$, where $f_j = f(x_j), j = 1, \cdots, J-1$,

The boundary condition of (2.1) is of the form $u_0^n = u_J^n = 0$.

The discrete form corresponding to (1.2) is:

$$\begin{aligned} &\frac{\Delta_+ \Delta_- u_j^*}{h^2} - \phi(u_j^*) + f_j = 0, \quad 0 < j < J \\ &u_0^* = u_J^* = 0 \end{aligned} \tag{2.2}$$

Let the discrete function u_h^n and u_h^* be the solution of difference equation (2.1) and (2.2) respectively. For $n = 0, 1, 2, \cdots$, the discrete function $v_h^n = \{v_j^n \mid j = 0, 1, \cdots, J\}$ is defined as $v_j^n = u_j^n - u_j^* (j = 0, 1, \cdots, J)$. Then v_h^n satisfies

$$\frac{v_j^{n+1} - v_j^n}{\Delta t} = \frac{\Delta_+ \Delta_- v_j^n}{h^2} - [(u_j^n)^3 - (u_j^*)^3] \tag{2.3}$$

for $j = 1, \cdots, J-1$ and $n = 0, 1, 2, \cdots$ Obviously, $v_0^n = v_J^n = 0, n = 0, 1, 2, \cdots$