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THE GLOBAL DUFORT-FRANKEL DIFFERENCE APPROXIMATION FOR NONLINEAR REACTION-DIFFUSION EQUATIONS*[1)]

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Abstract

In this paper, the initial value problem of nonlinear reaction-diffusion equation is considered. The Dufort-Frankel finite difference approximation for the long time scheme is given for the d-dimensional reaction-diffusion equation with the two different cases. The global solution and global attractor are discussed for the Dufort-Frankel scheme. Moreover properties of the solution are studied. The error estimate is presented in a finite time region and in the global time region for some special cases. Finally the numerical results for the equation are investigated for Allen-Cahn equation and some other equations and the homoclinic orbit is simulated numerically.

Keywords: globel Dufort-Frankel method, reaction-diffusion eqution, global attractor, error estimate, numerical experiments

1. Introduction

In this paper we consider the following initial-value problem of nonlinear reaction-diffusion equation:

$$\begin{cases} u_t = \gamma\Delta u - f(u); & (x,t) \in \Omega \times \mathbf{R}^+ \quad (1.1a) \\ u = 0, & x \in \partial\Omega \quad (1.1b) \\ u(x,0) = u_0(x), & x \in \Omega \quad (1.1c) \end{cases}$$

Here Ω is a bounded domain in $\mathbf{R}^d$ $(d \le 3)$ with a Lipschitz boundary $\partial\Omega$ and γ is a positive constant.

Let the set $\{|u^\star| : f(u^\star) = 0\}$ be not empty and $\bar{u} = \max\{|u^\star| : f(u^\star) = 0\}$.

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Assumption on the nonlinear function $f(u)$ is either (i). $rf(r) > 0$ for $|r| \geq \bar{u}$; or (ii). $f(-\bar{u}) = f(\bar{u})$.

Remark 1.1. Indeed, the assumption (ii) can be reduced as: there exist at least two different real roots for the nonlinear term $f(u)$ because we only need introduce a transform $v = u - (u_{\max} + u_{\min})/2$. Under this transform, the case (ii) holds for the new equation in which $v(x,t)$ is a new unknown function. Where $u_{\max}$ and $u_{\min}$ are the maximum and minimum roots of the nonlinear function $f(u)$.

The assumption (i) can be found in Hale[1]. Temam[2] who studied the global attractor for the equation (1.1) with the nonlinear term

$$f(s) = \sum_{j=0}^{2p-1} b_j s^j, \quad b_{2p-1} > 0.$$

In particular, the equation (1.1) which satisfies the condition (i) contains the Allen-Cahn equation[3] provided that $f(s) = \beta s(s^2 - 1)$. It is clear that the Allen-Cahn equation satisfies both assumptions (i) and (ii).

We also shall discuss the attractor for the case (ii) which the nonlinear function f satisfies (ii) but (i). For example, $f(s) = \beta(s^2 - 1)$, or $f(s) = \beta(|s| - 1)$. The nonlinear function $f(\cdot)$ which satisfy the assumptions(i) and (ii) are sketched on the Figures 1 and 2 respectively.

Fig.1 Case (i). $rf(r) > 0$ for $|r| \geq \bar{u}$

Fig.2 Case (ii). $f(\bar{u}) = f(-\bar{u})$

Throughout this paper, similar to discuss in Lu Bainian and Wan Guihua[4], we shall discuss the absorbing set and attractor of the discrete Dufort-Frankel finite difference equation of the equation (1.1) under condition (i) for any initial data and under (ii) for small initial data respectively. Moreover in section 2, we shall give some notations and some lemmas. In section 3, we shall study the global attractor for the finite difference equation and some properties of the difference equation. In section 4, we give the convergent theorem. Finally in section 5, we shall give several numerical examples to check our theoretical results which discussed in section 3. Furthermore, we study some properties for equation (1.1) with nonlinear term in the case (ii).

Elliott and Stuart[5] studied the Euler scheme for the equation (1.1). We know if we simulate the global solution or global attractor numerically to discuss the properties of the solution of (1.1), it will take a lots of CPU time on computer. So it is necessary to find a numerical scheme with large discrete step-lengthen. Indeed, Dufort-Frankel finite difference scheme is unconditionally stable for linear reaction-diffusion equation, but