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UNCONSTRAINED METHODS FOR GENERALIZED NONLINEAR COMPLEMENTARITY AND VARIATIONAL INEQUALITY PROBLEMS*1)

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Abstract

In this paper, we construct unconstrained methods for the generalized nonlinear complementarity problem and variational inequalities. Properties of the correspondent unconstrained optimization problem are studied. We apply these methods to the subproblems in trust region method, and study their interrelationships. Numerical results are also presented.

1. Introduction

Linear and nonlinear complementarity problems have many important applications in various fields such as economics, transportation etc., they have attracted much attention since early 1960's. A standard nonlinear complementarity problem is to find a $x \in R^n$ such that:

$$F(x) \geq 0,\ x \geq 0,\ x^T F(x) = 0, \tag{1.1}$$

where $F:\ R^n \longrightarrow R^n$. For simplicity, we often call it NCP. Many authors have studied this problem and encouraging results have been reported. One can find an excellently complete summary for it in [2]. For recent works, see [7], [9], [3], [8].

The generalized complementarity problem, denoted by $GCP(X, F)$, is to find a vector $x^* \in X$ such that:

$$F(x^*) \in X^*, \quad \text{and} \quad F(x^*)^T x^* = 0, \tag{1.2}$$

where X^* denotes the dual cone of X at x^*:

$$X^* = \{y \in R^n :\ y^T x \geq 0,\ \forall\ x \in X\}. \tag{1.3}$$

It is well known that, problem (**??**) is a special case of variational inequality problem, which takes the following form:

$$x^* \in X, \quad \text{and} \quad F(x^*)^T (y - x^*) \geq 0, \forall\ y \in X\ . \tag{1.4}$$

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For simplity, we called it $VI(X,F)$. But in general, a variational inequality problem does not equal to a complementarity problem. However, under certain conditions, a variational problem may be considered as a mixed nonlinear complementarity problem.

The purpose of this paper is to construct unconstrained method for (**??**) and (**??**). In the following section, we first describe some notations and concepts. Some results which will be used in this paper are also stated. In Section 3, we consider problem (**??**) as unconstrained optimization problem and study its optimal properties. The subproblem in trust region method is discussed in Section 4. We also explore the relations between them and show a new view of trust region method. Some numerical results are also reported in the last section.

2. Preliminaries

First, we give a definition which is due to [3]:

Definition 2.1. *We call a function $\phi : R^2 \to R$ NCP-function if it satisfies the nonlinear complementarity condition*

$$\phi(a,b) = 0 \Longleftrightarrow a \geq 0,\ b \geq 0,\ ab = 0.$$

Consider the function defined as follows:

$$\phi(a,b) = (\sqrt{a^2+b^2} - a)(\sqrt{a^2+b^2} - b), \quad (a,b) \in R^2. \tag{2.1}$$

It is obvious that it is a NCP-function. Furthermore, we have the following result[8]:

Lemma 2.1. *let $\phi(a,b)$ is defined by (**??**), the partial derivative of $\phi(a,b)$ equals to 0 if and only if (a,b) satisfies the complementarity condition. If (a,b) is strict complementarity, which means that $a+b>0$, we have:*

$$\frac{\partial^2\phi(a,b)}{\partial a^2} = 0, \quad \frac{\partial^2\phi(a,b)}{\partial a\partial b} = 0, \quad \frac{\partial^2\phi(a,b)}{\partial b^2} = 1 \tag{2.2}$$

hold for $b=0$ and $a>0$, and

$$\frac{\partial^2\phi(a,b)}{\partial a^2} = 1, \quad \frac{\partial^2\phi(a,b)}{\partial a\partial b} = 0, \quad \frac{\partial^2\phi(a,b)}{\partial b^2} = 0 \tag{2.3}$$

hold for $a=0$ and $b>0$.

Karamardian[4] first established the following basic relation between $GCP(X,F)$ and $VI(X,F)$.

Theorem 2.1. *Let X be a convex cone. Then $x^* \in X$ solves the problem VI(X,F) if and only if x^* solves the GCP(X,F).*

In the case where the set X is defined by the inequalities of the form

$$X = \{x \in R^n : g_i(x) \leq 0, i = 1,2,\ldots,m;\ h_j(x) = 0, j = 1,2,\ldots,p\}, \tag{2.4}$$