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# A DUAL COUPLED METHOD FOR BOUNDARY VALUE PROBLEMS OF PDE WITH COEFFICIENTS OF SMALL PERIOD*1)

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**Abstract**

In this paper the homogenization method is improved to develop one kind of dual coupled approximate method, which reflects both the macro-scope properties of whole structure and its loadings, and micro-scope configuration properties of composite materials. The boundary value problem of woven membrane is considered, the dual asymptotic expression of the exact solution is given, and its approximation and error estimation are discussed. Finally the numerical example shows the effectiveness of this dual coupled method.

## 1. Introduction

The mechanical performance analysis of the structures made of woven composite material and periodically perforated material is often encountered in the modern engineering analysis. Since this kind of composite material has periodically basic configurations, the static analysis of the structures made from this composite material leads to the boundary value problem of elliptic PDE with periodic coefficients, for example, the equilibrium problem of woven membrane under traverse loadings can be expressed in the boundary value problem of two dimension two order elliptic PDE as follows:

$$\text{(P)}\qquad \begin{cases} -\dfrac{\partial}{\partial x_i}(a^{\epsilon}_{ij}(x)\dfrac{\partial u_\epsilon}{\partial x_j}) = f(x), x\in\Omega \\ u_\epsilon|_{\partial\Omega} = 0 \end{cases}$$

$\Omega$ is shown in Figure 1, $x$ represents both global coordinates of the structure and macro-scope properties of its geometry and loadings, $\epsilon$ is the length of basic configuration of composite material which is shown in Figure 2, and $a^{\epsilon}_{ij}(x)$ has periodicity, symmetry and ellipticity. Let $y=\dfrac{x}{\epsilon}$ and $a_{ij}(y)=a^{\epsilon}_{ij}(x)$, and then $a_{ij}(y)$ has periodicity with

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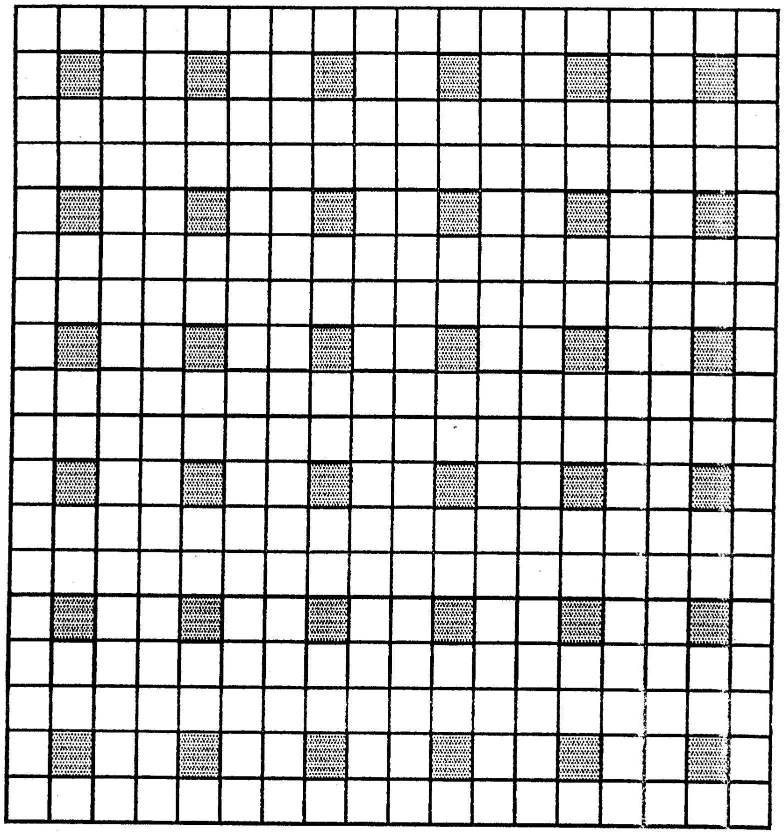

Figure 1. Weaved membrane

length 1. Engineers are often concerned with the stress state in some basic configuration due to stress concentration, and then most of breakages of structures happen locally. In order to obtain accurate stress results in the basic configuration the whole structure must be partitioned into very small meshes using finite element method, this leads to very large scale computation.

For this kind of problems of elliptic PDE, A. Bensoussan, J.L. Lions and G. Papanicolaou[1] proposed one kind of homogenization methods. The solution $u_\epsilon(x)$ of problem $(P)$ is asymptotically expanded in dual $(x, y)$ form as follows:

$$u_\epsilon(x) = u_0(x) + \epsilon u_1(x, y) + \epsilon^2 u_2(x, y) + \cdots, \tag{1.1}$$

where $u_0(x)$ is called as homogenization solution, and represents global mechanical and physical properties of structure, and $u_i(x, y)$ reflects both global mechanical behavior and the effect of micro-configuration of composite material. Formally the solution $u_\epsilon(x)$ is considered as one homogenization solution plus a series of relative periodic functions with high order coefficients $\epsilon^i$.

In [1] the main results of homogenization method achieved are following: