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TRACE AVERAGING DOMAIN DECOMPOSITION METHOD WITH NONCONFORMING FINITE ELEMENTS*

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Abstract

We consider, in this paper, the trace averaging domain decomposition method for the second order self-adjoint elliptic problems discretized by a class of nonconforming finite elements, which is only continuous at the nodes of the quasi-uniform mesh. We show its geometric convergence and present the dependence of the convergence factor on the relaxation factor, the subdomain diameter H and the mesh parameter h. In essence, this method is equivalent to the simple iterative method for the preconditioned capacitance equation. The preconditioner implied in this iteration is easily invertible and can be applied to preconditioning the capacitance matrix with the condition number no more than $O\big((1+\ln\frac{H}{h})max(1+H^{-2},1+\ln\frac{H}{h})\big)$.

1. Introduction

Domain decomposition refers to numerical methods for obtaining solutions of scientific and engineering problems by combining solutions to problems posed on physical subdomains, or, more generally, by combining solutions to appropriately constructed subproblems. It has been a subject of intense interest recently because of its suitability for implementation on high performance computer architectures. Some papers are listed in the references herein, which indicate that much progress has been made in the study of nonoverlap domain decomposition methods , also known as the substructuring methods. It is rather complicated in the case of multi-subdomains with the internal cross points. A cross point is defined to be the common boundary point of more than two subdomains. With the techniques of the separation of the internal cross points from other mesh nodes, Bramble et al.[2,3,4,5], Widlund[18], constructed different preconditioners for the algebraic system of equations which arise from the following self-adjoint elliptic problems via conforming finite element methods:

$$u\in H_0^1(\Omega):\ a(u,v)=(f,v),\quad \forall\ v\in H_0^1(\Omega), \tag{1.1}$$

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where

$$a(u,v)=\int_\Omega\Big[\sum_{i,j=1}^{2}a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}+a_0(x)uv\Big],\quad (f,v)=\int_\Omega fv, \tag{1.2}$$

$\Omega\subseteq\Re^2$ is a bounded polygonal open domain, $f\in H^{-1}(\Omega)$, $a_0(x)\geq 0$, $a_{ij}(x)$, $i,j=1,2$, $a_0(x)$ are piecewise smooth and bounded functions in Ω, (a_{ij}) is a symmetric, uniformly positive definite matrix in Ω. All their preconditioners can be inversed easily in parallel and precondition the stiff matrix with the condition number no more than $O((1+\ln\frac{H}{h})^2)$, where H,h are the subdomain diameter and the fine mesh parameter, respectively. Bourgat et al.[1] introduced an iterative substructuring method with the trace averaging operator to deal with the internal cross points, and illustrated its efficiency in the conforming discrete case with plenty of numerical experiments. Later, Chu[9] gave the theoretical proof of its convergence.

The present paper is concerned with the construction of efficient iterative schemes for solving (1.1) discretized by a class of nonconforming finite elements, which is only continuous at the mesh nodes. Let $\Omega_h=\{e\}$ be a quasi-uniform mesh of Ω, where h is the mesh parameter and e, a triangle or a quadrilateral, represents typical element in Ω_h. Let the nonconforming finite element space

$$\begin{aligned}S^h(\Omega)=&\{v_h:\ \ v_h=\theta_h+w_h,\ \theta_h\in T^h(\Omega),\ w_h(x)=0,\ \forall\text{ node }x\in\overline{\Omega},\\ &\quad w_h|_e\text{ is a finite order polynomial, }\forall\ e\in\Omega_h\},\end{aligned}$$

where

$$T^h(\Omega)=\{\theta_h\in C(\Omega):\ \theta_h|_e\text{ is linear (bilinear) if }e\text{ is a triangle (quadrilateral), }\forall\, e\in\Omega_h\}.$$

Here, a node $x\in\overline{\Omega}$ is defined to be the vertex of some $e\in\Omega_h$. In practice, there are many nonconforming finite elements which are only continuous at the mesh nodes, e.g. Wilson elements[19], triangle membrane elements[8], etc. Denote

$$\begin{aligned}S_0^h(\Omega)&=\{v_h\in S^h(\Omega):\ v_h(x)=0,\ \forall\text{ node }x\in\partial\Omega\},\\ A(u,v)&=\sum_{e\subset\Omega}\int_e\Big[\sum_{i,j=1}^{2}a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}+a_0uv\Big].\end{aligned}$$

Then, the nonconforming finite element discrete problem for (1.1) is

$$u_h\in S_0^h(\Omega):\ A(u_h,v_h)=(f,v_h),\ \forall\ v_h\in S_0^h(\Omega). \tag{1.3}$$

In the two-subdomain nonoverlap cases, Gu[12] proposed and analysed a series of algorithms for solving (1.3) via the extension theorem for nonconforming elements[13]. In the multi–subdomain nonoverlap cases, many preconditioners for (1.3) have been constructed successfully, based on the conforming interpolation operator and the essential estimates[12,9]. All are as efficient as their counterparts in the conforming discrete cases. Furthermore, we note that a hierarchical basis multilevel method with