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MODELS OF ASYNCHRONOUS PARALLEL NONLINEAR MULTISPLITTING RELAXED ITERATIONS*

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Abstract

In the sense of the nonlinear multisplitting and based on the principle of sufficiently using the delayed information, we propose models of asynchronous parallel accelerated overrelaxation iteration methods for solving large scale system of nonlinear equations. Under proper conditions, we set up the local convergence theories of these new method models.

1. Introduction

Consider the large scale system of nonlinear equations

$$F(x)=0, \qquad F:\mathcal{D}\subset R^n\to R^n. \tag{1.1}$$

Given α ($\alpha\le n$, an integer) nonempty subsets $J_i(i=1,2,\cdots,\alpha)$ of the set $\{1,2,\cdots,n\}$ with

$$\bigcup_{i=1}^{\alpha} J_i=\{1,2,\cdots,n\},$$

where $J_1,J_2,\cdots,J_\alpha$ may overlap among them. For $i=1,2,\cdots,\alpha$, we assume that

a) $f^{(i)}:\mathcal{D}\times\mathcal{D}\subset R^n\times R^n\to R^n$ satisfies

$$f^{(i)}(x;x)=(f_1^{(i)}(x;x),f_2^{(i)}(x;x),\cdots,f_n^{(i)}(x;x))^T=F(x), \qquad \forall x\in\mathcal{D};$$

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b) $E_i = \mathrm{diag}(e_1^{(i)}, e_2^{(i)}, \cdots, e_n^{(i)}) \in L(R^n)$ satisfies

$$\begin{cases} e_m^{(i)} = \begin{cases} e_m^{(i)} \geq 0, & \text{for } m \in J_i \\ 0, & \text{for } m \notin J_i \end{cases}, \quad m = 1, 2, \cdots, n \\ \sum_{i=1}^{\alpha} E_i = I \quad (I \in L(R^n) \text{ being identity matrix}). \end{cases}$$

Then, the collection of pairs $(f^{(i)}, E_i)(i = 1, 2, \cdots, \alpha)$ is called a nonlinear multisplitting of the mapping $F : \mathcal{D} \subset R^n \to R^n$.

Nowadays, there have been a lot of more deepened research results on both the parallel methods, designed by making use of this concept for solving the system of nonlinear equations (1.1) on the high-speed multiprocessor systems, and their theory analyses[1−4]. Considering the intrinsic shortcomings of the synchronous parallel methods, the parallel iterative methods suitable to the asynchronous computational environments are particularly considerable. It was just in the sense of the nonlinear multisplitting that paper [4] set up a class of asynchronous parallel AOR iterative methods for solving the system of nonlinear equations (1.1).

Based on the already existed results, in this paper we propose a class of method models of asynchronous parallel accelerated overrelaxation iterations for solving the system of nonlinear equations (1.1) by making use of the above concept of nonlinear multisplitting and in light of the principle of sufficiently using the delayed information. These method models give consideration to both the advantages of the nonlinear multiple splittings and the concrete characterizations of the multiprocessor systems, and are of a lot of good behaviours such as convenient computations, flexible and freed communications and so on. Therefore, they can greatly execute the efficiency of practical computations of the multiprocessor systems. Following different choices of the relaxation parameters, not only can the convergence properties of the new asynchronous parallel relaxation method models be improved, but also many applicable and efficient asynchronous parallel nonlinear multisplitting relaxed iteration methods such as the Jacobi, Gauss-Seidel, SOR and so on can be obtained. Meanwhile, the asynchronous parallel nonlinear multisplitting AOR-Newton, -Chord, -Steffensen programs, being of highly practical value, are set up, which makes the new method models become further more convenient, applicable and effective in concrete implementations. Under suitable conditions, we establish local convergence theories for the new models of the asynchronous parallel nonlinear multisplitting relaxed methods, and estimate the asymptotic convergence rates of them, too. At last, the local convergence properties of the asynchronous parallel nonlinear multisplitting AOR-Newton, -Chord, -Steffensen programs are discussed in detail in a unified form.

This work is really developments of the results shown in paper [2], and is also further improvements and generalizations of those in paper [4].