Journal of Computational Mathematics, Vol.13, No.3, 1995, 211-217.

DETERMINATION AND CORRECTION OF AN INCONSISTENT SYSTEM OF LINEAR INEQUALITIES *1)

Y.Y. Nie
(*Laboratory of CAD/CAM Technology for Advanced Manufacturing , Academia Sinica, Shenyang, China*)
S.R. Xu
(*Department of Computer Science, Zhongshan University, Guangzhou, China*)

Abstract

In this paper the problems to determine an inconsistent system of linear inequalities and to correct its right-hand side vector are solved by using the isometric plane method for linear programming. As an example, the suitable perturbation quantity of the perturbed inequalities of ill-conditioned linear equations is determined in the numerical experiments.

1. Introduction

In mathematical models of some practical problems, such as constraint conditions of linear programming problems, the systems of linear inequalities which should be consistent may be inconsistent due to incorrect input data. It is sometimes difficult or even impossible to obtain better data. So the problems to determine and correct an inconsistent system of linear inequalities become significant.

In 1988, R.L. Mogilevskaya and P.A. Shvartsman published an algorithm to correct the right-hand side vector of inconsistent system in such a way that the new system is consistent system and is not too far from the model. The algorithm is mathematically simple and give the user the possibility to choose a correction suitable with respect to the corresponding practical problem[1].

Consider the following system of linear inequalities

$$AX > B, \tag{1.1}$$

where $A = (a_{i,j})$ is an $m \times n$ matrix $(m \geq 1, n \geq 2)$, $X = (x_1, x_2, \cdots, x_n)^T$ and $B = (b_1, \cdots, b_m)^T$ are $n-$ and $m-$ dimensional vectors respectively, $(\cdot)^T$ denotes transpose of $(\cdot)$. Note that no equality is contained within (1.1) by means of appropriate treatment.

* Received December 8, 1992.
1) This Project is Supported in part by National Natural Science Foundation of China and by Foundation of Zhongshan University Advanced Research Center.

The system (1.1) is said to be nonreducibly inconsistent if it is inconsistent but each of its proper subsystems is consistent . In [1], the main idea to correct an inconsistent system is to select nonreducibly inconsistent subsystems of the given system to perform corrections on these subsystems. For each nonreducibly inconsistent system, the set of corrections of its right- hand side vector making the system consistent may be described by a unique formula. In order to select a nonreducibly inconsistent subsystem from (1.1) the following series of LP problems must be solved

$$-\varepsilon \to \max, \quad A_k X > B_k, \quad a_{k+1}X + \varepsilon > b_{k+1} \geq 0, \tag{1.2}$$

where $A_k X > B_k$ is a consistent subsystem of (1.1) and $a_{k+1}X > b_{k+1}$ is an inequality, which does not belong to $A_k X > B_k$, but belongs to (1.1).

There is a more suitable method to determine the inconsistent system (1.1) and to correct its right-hand side vector, in authors' opinion, that is the isometric plane method for linear programming[2] which has been presented in 1988.

In the isometric plane method so-called general LP problem,

$$C^T X \to \max, \quad AX > B \tag{1.3}$$

is considered, here the constraint set is the same with (1.1) formally. The constraint set of (1.3) can form arbitrary convex polyhedron in $n-$dimensional space, specially it can be empty set that is equivalent to inconsistent of (1.1) or no solution of (1.3). In [2] a way to determine the consistence of (1.1) without considering the selection of nonreducibly inconsistent subsystems are provided. However, the isometric plane method for linear programming is also suitable to select nonreducibly inconsistent subsystems with an inconsistent system in an alternative way which will be discussed in section 2.

2. Selection of Nonreducibly Inconsistent Subsystems

In the system (1.1) an initial consistent subsystem is easy found, for example, the first inequality

$$a_1 X = (a_{11}, a_{12}, \cdots, a_{1n}) \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} > b_1,\ n \geq 2,\ a_1 \neq 0$$

is clearly consistent and there are innumerable points, which are called interior points, to satisfy

$$a_1 X > b_1.$$

Without loss of generality , assume that

$$A_k X = \begin{bmatrix} a_1 \\ \vdots \\ a_k \end{bmatrix} X > B_k = \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix}, \quad 1 \leq k < m \tag{2.1}$$