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SIMPLIFIED ORDER CONDITIONS OF SOME CANONICAL DIFFERENCE SCHEMES[*1)]

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Abstract

The main purpose of this paper is to develop and simplify the general conditions for an s-stage explicit canonical difference scheme of q-th order, while the simplified order conditions for canonical RKN methods which are applied to a special kind of second order ordinary differential equations are also obtained here.

1. Introduction

In [5–8], explicit canonical difference schemes up to the fourth order are constructed for separable Hamiltonian systems (i.e., systems with the Hamiltonian function $H(p,q)=U(p)+V(q)$). But unfortunately, we can not find the general order conditions for this method whether sn algebraic or Lie method is used to get order conditions for some scheme of a definite stage number. In this paper, we will use P-series introduced in [4] and tree methodology used by Sanz-Serna in [2] to get the general order conditions for the explicit canonical method and then simplify these conditions to get much more independent ones.

In [12], we have already omitted some redundant order conditions for canonical RKN methods, but there are still some order conditions dependent on each other because of the canonicity of the methods. In this paper, we will drop out these order conditions and get much simpler ones.

In Section 1, we give some definitions and notations about graphs and trees; they are the basis of understanding the later derivation in Sections 2 and 3. Section 2 is about general order conditions of canonical explicit methods and their simplified form. In Section 3, we get simplified order conditions of the canonical RKN method.

1. Graphs and Trees

In this section, we only give some definitions and notations about graphs and trees which will be used in this paper. For details about graphs and trees, one can refer to [2],[4].

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1. Graphs. Let n be a positive integer. A **graph** G of **order** n is a pair $\{V, E\}$ formed by a set V with $\text{Card}(V) = n$ and a set E of un-ordered pairs (v, w), with $v, w \in V, v \neq w$, which may be empty. The elements of V and E are called **vertices** and **edges** of the graph respectively. Two vertices v, w are called **adjacent** if $(v, w) \in E$.

Graphs can be represented graphically as Fig. 1 shows. In Figure 1, the black dots represent the vertices of the graph, and the lines joining the dots are the edges.

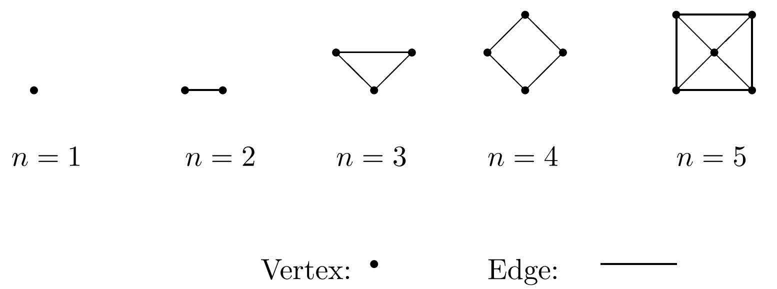

Fig. 1. Graphs

Giving the vertices of G an arbitrary set of labels, we then get a **labeled graph** $g, g \in G$. By labeling the graph G in different ways, we can get different labeled graphs. For convenience, we often use letters $i, j, k, l, \cdots$ as the labels in this paper. Notice that in the definition of the graph G, we use v and w to denote two different vertices; they are not the labels of these vertices. Fig. 2 shows a graph of order 4 and its different labelings.

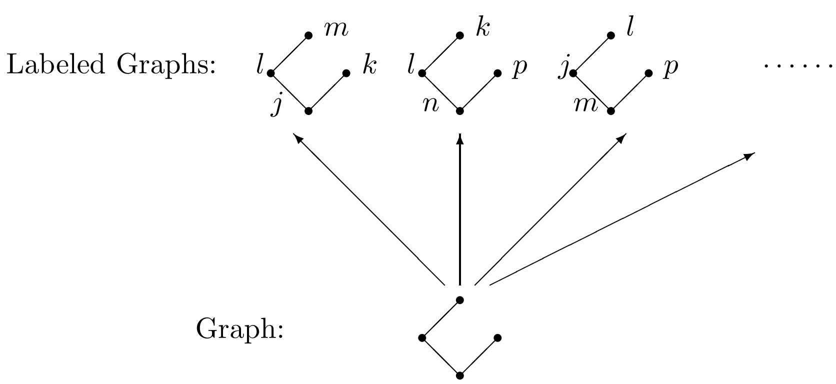

Fig. 2. Graphs and Labeled Graphs

Now we consider two kinds of special graphs: P-graphs and S-graphs.

A **P-graph** PG is a special graph which satisfies:

i) its vertices are divided into two classes: **“white”** and **“black”**;

i) the two adjacent vertices of a PG cannot be of the same class.