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A SIMPLIFIED VISCOSITY SPLITTING METHOD FOR SOLVING THE INITIAL BOUNDARY VALUE PROBLEMS OF NAVIER-STOKES EQUATION*1)

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Abstract

Based on the approximation of the linear operator semigroup, this paper proposes a simplified viscosity splitting method for solving the initial boundary value problems of the N-S equation. Some stability and convergence estimates of the method are proved. In particular, the mechanism of Chorin's method is explained and justified by the splitting method.

§1. Introduction

In this paper, the following initial boundary value problem of the Navier-Stokes (N-S) equation is considered

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \frac{1}{\rho}\nabla p = \nu\Delta u + f, \quad (x,t) \in \Omega \times [0,T), \tag{1.1}$$

$$\operatorname{div} u = \nabla \cdot u = 0, \qquad (x,t) \in \Omega \times [0,T), \tag{1.2}$$

$$u(x,t)|_{x\in\partial\Omega} = 0, \qquad t \in [0,T), \tag{1.3}$$

$$u(x,0) = u_o(x), \qquad x \in \Omega \tag{1.4}$$

where Ω is assumed to be a simply connected and bounded domain in $I\!R^2, u = (u^1(x,t), u^2(x,t))^T$ is the velocity, $p = p(x,t)$ the pressure and $f = (f^1(x,t), f^2(x,t))^T$ the body force, constants $\rho, \nu > 0$ are the density and viscosity respectively. Re$= \frac{1}{\nu}$ represents Reynold's number.

Various viscosity splitting methods have been developed for solving the N–S equation. A remarkable one was proposed by Chorin in 1973 (see [3]) for calculating

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viscous incompressible flows with high Reynold's number. In terms of this method, the nonstationary N-S equation is solved by alternatively solving the Euler equation and the Stokes equation while the first equation by the characteristic vortex blob method and the next one by the random walk method, and in order to fulfil the no-slip condition $u \cdot \tau=0$ on the boundary $\partial\Omega$, vortex sheet is introduced to modify the solution obtained. A great deal of calculation has demonstrated that Chorin's method is effective for flows with high Reynold's number. However, the theoretical proof of convergence for Chorin's method has not been done yet, except for the case of pure initial value problem.

Recently, Ying Lungan has devoted several papers to the viscosity splitting method for the N–S equation in a bounded domain (see [4], [5]). He proposed a new viscosity splitting method of semidiscrete form with a special projection operator in it, and succeeded in proving the convergence and deriving an error estimate for his method under the assumption that the solution of the problem is sufficiently smooth. In [4], he gives a mathematical formulation for Chorin's method, and based on this formulation, he points out that "Chorin's scheme would cause divergence", i.e. the approximative solution obtained from Chorin's method does not converge to the solution of the original N-S equation. Since satisfactory results of Chorin's method have been shown in practice, a further discussion on the interpretation and the justice of Chorin's method might be valuable.

In this paper, making use of the approximation of the linear operator semigroup, we first present another viscosity splitting method with no projection operator like that in Ying's method. Then we prove the stability and convergence estimates for this method. Finally, based on these theoretical analyses, we give a new interpretation and justification to Chorin's method.

§2. A Simplified Viscosity Splitting Method

We first recall some concepts related to the semigroup theory of the N-S equation in two dimensions. Note that finding the projection in step 2 of the viscosity splitting method in [4] is equivalent to solving a boundary value problem of the biharmonic equation, so there is some computational complexity in doing it. In order to simplify the computation, we introduce a simplified viscosity splitting method of the N–S equation in a bounded domain.

2.1 Definitions and Concepts Related

Define subspaces

$$X = \text{ the closure of} \{u \in (C_0^\infty(\Omega))^2;\ \operatorname{div} u = 0)\} \text{ in } (L^2(\Omega))^2,$$