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REGULARIZED TWO-STAGE STOCHASTIC VARIATIONAL INEQUALITIES FOR COURNOT-NASH EQUILIBRIUM UNDER UNCERTAINTY*

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Abstract

A convex two-stage non-cooperative multi-agent game under uncertainty is formulated as a two-stage stochastic variational inequality (SVI). Under standard assumptions, we provide sufficient conditions for the existence of solutions of the two-stage SVI and propose a regularized sample average approximation method for solving it. We prove the convergence of the method as the regularization parameter tends to zero and the sample size tends to infinity. Moreover, our approach is applied to a two-stage stochastic production and supply planning problem with homogeneous commodity in an oligopolistic market. Numerical results based on historical data in crude oil market are presented to demonstrate the effectiveness of the two-stage SVI in describing the market share of oil producing agents.

Mathematics subject classification: 90C15, 90C33.
Key words: Two-stage stochastic variational inequalities, Cournot-Nash equilibrium, Regularized method, Progressive hedging method, Uncertainty, Oil market share.

1. Introduction

We consider a two-stage non-cooperative multi-agent game under uncertainty. The model is used to describe a homogenous commodity production and supply planning problem in an oligopolistic market. In particular, we focus on a J-player stochastic Nash equilibrium problem (SNEP) of Cournot competition, whose solution concept is characterized by stochastic Cournot-Nash (CN) equilibria. Conventional non-cooperative game theory has a long history being an effective model to describe market behaviour, yet mostly under deterministic settings. In order to explore characteristics of real markets, one cannot neglect the presence of uncertainty.

Researchers have studied various real markets through SNEPs in recent years. Jofré, Rockafellar and Wets [24] investigated various economic equilibria using SVIs. A scenario-based multi-stage oligopolistic market equilibrium problem under uncertainty was discussed in [16]. A two-settlement oligopolistic equilibrium with uncertainty in the future market was presented in [48]. For practical applications, electricity markets with hydro-electric distribution have been studied by Philpott, Ferris and Wets [30] in which the levels of water reserves were modelled under uncertainty. The contemporary treatment of classical equilibrium problems are investigated through finite-dimensional variational inequalities (VIs) and complementarity problems (CPs) with a wide range of applications under the assumption of deterministic and single-stage

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decision, see [14, 20] and references therein. Yet, many practical applications are formulated under uncertainty [9, 10, 21, 28, 29, 47], and the corresponding multi-stage SVIs and CPs were studied extensively over the last two decades. In [5], Chen and Fukushima considered the stochastic linear CP (LCP) by expected residual minimization (ERM) procedure. The quasi-Monte Carlo method was adopted to generate scenarios of observation and thus to obtain the discrete approximation problem. Chen, Wets and Zhang [10] investigated SVI problems by the ERM procedure, and the sample average approximation (SAA) method was employed to approximate the expected smoothing residual function. More recently, as an extension from single-stage to multi-stage decision processes, Rockafellar and Wets [40] put forward multi-stage SVI problems, which laid a solid foundation for further research. In [6], Chen, Pang and Wets introduced the ERM procedure for two-stage SVI problems and the Douglas-Rachford splitting method was used to present numerical results. Chen, Sun and Xu [9] considered a two-stage stochastic LCP. Structural properties of the problem were studied under the assumption of strong monotonicity, and a discrete scheme was conducted by partition of the support set and the corresponding convergence assertion was established. More generally, Chen, Shapiro and Sun [8] investigated the SAA of two-stage stochastic nonlinear generalized equations, which included two-stage nonlinear SVI problems as special cases. Exponential rate of convergence was derived by using the technique of perturbed partial linearization. From the perspective of the numerical calculation, the equilibrium problems are usually rewritten as a minimization problem, mostly nonsmooth. For this class of problems, the smoothing techniques (see [3]) can be employed so that differentiable methods, e.g., Newton's method, become applicable in solving the smoothing problem, see for instance [4, 11–13]. For numerical implementation, Rockafellar and Sun extended the well-known progressive hedging method (PHM) for multi-stage stochastic programming problems to multi-stage SVIs in [33].

The decision vectors of production and supply plan problems, so-called *strategies*, are distinguished into two categories: (i) those of "here-and-now" type, which do not depend on outcomes of random events in the future, and (ii) those treated as responses. The goal of this paper is to establish a model that describes the market mechanism for Cournot competition under uncertainties. The solution concepts of the model assemble the real strategies adopted by participants of the market. We will provide methodology to solve the proposed model and demonstrating its numerical implementation.

We summarize the main contributions of this paper as follows.

- A two-stage stochastic Nash equilibrium problem is proposed to model production and supply competition of a homogenous product under uncertainty in an oligopolistic market.
- The model is recast as two-stage SVIs whose solutions characterize a CN equilibrium.
- A regularized sample average approximation method is proposed to solve the two-stage SVIs with convergence properties under mild assumptions.
- The model is tested numerically for its effectiveness. Moreover, it is used to describe the market share observation in the world market of crude oil. We show that our model is not only able to reproduce historical in-sample market share but also capable of making out-of-sample predictions based on real data sets.

The remaining of the paper is organized as follows. In Section 2, the two-stage SNEPs are developed and recast into two-stage SVIs. Section 3 contains structural analysis of two-stage