

# THREE-DIMENSIONAL GRAVITY-MAGNETIC CROSS-GRADIENT JOINT INVERSION BASED ON STRUCTURAL COUPLING AND A FAST GRADIENT METHOD\*

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## Abstract

In order to effectively solve the low precision problem of the single gravity density inversion and the magnetic susceptibility inversion, and the limitation of the gravity-magnetic joint inversion method based on the petrophysical parameter constraint, this paper studies the three-dimensional gravity-magnetic cross-gradient joint inversion based on the structural coupling and the fast optimization algorithm. Based on the forward and inversion modeling of three-dimensional gravity density and three-dimensional magnetic susceptibility using the same underground grid, along with cross-gradient coupling as the structural constraint, we propose a new gravity-magnetic joint inversion objective function including the data fitting term, the total variation regularization constraint term and the cross-gradient term induced by the structural coupling. The depth weighted constraint and the data weighting constraint are included into the objective function, which requires different physical property models to minimize their respective data residuals. At the same time, the cross-gradient term tends to zero, so that the structure of the gravity and magnetic models tends to be consistent. In realization, we address a fast and efficient gradient algorithm to iteratively solve the objective function. We apply this new joint inversion algorithm to the 3D gravity-magnetic model inversion test and compare it with the results of a single inversion algorithm. The experimental tests of synthetic data indicate that the gravity-magnetic cross-gradient joint inversion method can effectively improve the accuracy of the anomaly position and numerical accuracy of the inverted anomaly physical parameters compared with the single physical inversion method.

*Mathematics subject classification:* 86-08, 65J20, 65K10.

*Key words:* Joint inversion, Gravity, Magnet, Cross-gradient, Regularization.

## 1. Introduction

Gravimetric and magnetic explorations are two effective methods in geophysical exploration. Both methods are volume exploration and can be performed quickly and economically. The gravimetric method is widely used to dig out about the deep structure of the earth, to divide the structural units, and to detect minerals. The magnetic method also plays an important role in many aspects, a typical application is to find non-ferrous metal minerals. In the gravimetric and magnetic exploration, it is necessary to invert the distribution of the underground anomalies

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according to the data to guide the follow-up work reasonably and effectively. Advances in the gravimetric and magnetic techniques play a crucial role in the exploration technology, as the inversion results can indicate important information such as the distribution of subsurface density (or susceptibility) and the shape of the model. However, the geophysical exploration targets which we are facing are much more complicated and difficult than before (such as complex surface conditions and underground structures), making it difficult to obtain high-quality data from geophysical methods such as gravity strategies [31]. In addition, different geophysical exploration methods have their own properties for the imaging of media underground. Especially for the gravimetric and magnetic exploration, the most important problem for the inversion of the field data is the inherent ill-posedness [19,20]. There are many reasons for the ill-posedness of the inverse problem, including: different anomalous bodies can correspond to the same observation data, insufficient observation data, environmental interference and observation errors caused by limited instrument accuracy [19,20]. To reduce the ill-posedness of the inverse problem and improve the accuracy of inversion results, joint inversion method become a tendency in geophysics [7, 8, 12, 14, 23]. Joint inversion methods, which comprehensively using the information obtained by various exploration methods, can effectively depict the underground geological structure.

The joint inversion strategy between different physical parameters can be divided into two categories. One is the coupling method of petrophysical parameters, which is a joint inversion by establishing an empirical relationship between two or more physical parameters [18]. For example, based on the empirical relationship between density and velocity, Jegen et al. in [18] developed a joint inversion method for gravity, magnetism, and seismology. However, in most cases, the empirical relationship between these physical parameters depends on the specific geological conditions; therefore, the limitations of this kind of method are relatively strong. The other type of method belongs to the model structure coupling method, which performs joint inversion by finding the structural consistency of two or more physical models [6, 15, 22, 26, 27]. Among the joint inversion methods, the representative method is the cross-gradient joint inversion method based on the structural coupling proposed by Gallardo and Meju in [7, 8]. This method achieved satisfactory results in the joint inversion of seismic travel time and direct current (DC) resistivity method. This approach is to make their structures tend to be consistent by the cross-gradient values between the models tending to zero. This method neither depends on the empirical relationship between physical parameters, nor on specific geological conditions, and has received extensive attention [4, 10, 12, 14, 23, 25, 35]. For example, Moorkamp et al. in [23] proposed a joint inversion framework for three-dimensional MT, gravity and seismic refraction data based on cross-gradients; Abubakar et al. in [1] combined cross-gradient method with petrophysical method to realize surface wave and magnetotelluric joint inversion; Fregoso et al. in [5] combined Euler deconvolution with cross-gradient and applied it to gravity-magnetic joint inversion, which achieved good results; Gao et al. in [13] developed an efficient 3D seismic travel time and DC resistivity joint inversion strategy based on the cross-gradient structure constraint, their synthetic model tests showed that the joint inversion imaging results improve the resolution of the solution compared with single physical property imaging.

In order to effectively solve the problem of the low precision of the single physical property imaging method, e.g., the gravity inversion or the magnetic susceptibility inversion, and the limitation of the gravity-magnetic joint inversion method based on the petrophysical parameter constraint, in this paper, a three-dimensional gravity-magnetic cross-gradient joint inversion